Published for SISSA by Springer

Received: *December 3, 2014*
Accepted: *January 31, 2015*
Published: *February 17, 2015*

Correlation functions in ω-deformed $\mathcal{N} = 6$ supergravity

A. Borghese,[a] **Y. Pang,**[a] **C.N. Pope**[a,b] **and E. Sezgin**[a]

[a] *George P. & Cynthia Woods Mitchell Institute for Fundamental Physics and Astronomy, Texas A&M University, College Station, TX 77843, U.S.A.*

[b] *DAMTP, Centre for Mathematical Sciences, Cambridge University, Wilberforce Road, Cambridge CB3 OWA, U.K.*

E-mail: andrea.borghese@virgilio.it, pangyi1@physics.tamu.edu, pope@physics.tamu.edu, sezgin@physics.tamu.edu

Abstract: Gauged $\mathcal{N} = 8$ supergravity in four dimensions is now known to admit a deformation characterized by a real parameter ω lying in the interval $0 \leq \omega \leq \pi/8$. We analyse the fluctuations about its anti-de Sitter vacuum, and show that the full $\mathcal{N} = 8$ supersymmetry can be maintained by the boundary conditions only for $\omega = 0$. For non-vanishing ω, and requiring that there be no propagating spin $s > 1$ fields on the boundary, we show that $\mathcal{N} = 3$ is the maximum degree of supersymmetry that can be preserved by the boundary conditions. We then construct in detail the consistent truncation of the $\mathcal{N} = 8$ theory to give ω-deformed SO(6) gauged $\mathcal{N} = 6$ supergravity, again with ω in the range $0 \leq \omega \leq \pi/8$. We show that this theory admits fully $\mathcal{N} = 6$ supersymmetry-preserving boundary conditions not only for $\omega = 0$, but also for $\omega = \pi/8$. These two theories are related by a U(1) electric-magnetic duality. We observe that the only three-point functions that depend on ω involve the coupling of an SO(6) gauge field with the U(1) gauge field and a scalar or pseudo-scalar field. We compute these correlation functions and compare them with those of the undeformed $\mathcal{N} = 6$ theory. We find that the correlation functions in the $\omega = \pi/8$ theory holographically correspond to amplitudes in the $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ ABJM model in which the U(1) Noether current is replaced by a dynamical U(1) gauge field. We also show that the ω-deformed $\mathcal{N} = 6$ gauged supergravities can be obtained via consistent reductions from the eleven-dimensional or ten-dimensional type IIA supergravities.

Keywords: AdS-CFT Correspondence, Supergravity Models

ArXiv ePrint: 1411.6020

Article funded by SCOAP[3].

doi:10.1007/JHEP02(2015)112

JHEP02(2015)112

Contents

1 Introduction

For thirty years after its construction in 1982 [1], the SO(8) gauged maximally supersymmetric $\mathcal{N} = 8$ supergravity was widely considered to be a unique theory. Interestingly, using the embedding tensor formulation [2], it was recently realized that there is a one-parameter extension of the theory, commonly denoted by ω, associated with a mixing of the electric and magnetic vector fields employed in the SO(8) gauging [3, 4]. Inequivalent $\mathcal{N} = 8$ theories are parameterised by ω in the range $0 \leq \omega \leq \pi/8$. This development has raised numerous interesting questions, such as its possible higher-dimensional string/M theory origin and the consequences of the ω deformation for the holographic dual theory.

In this paper, as a step towards addressing the holography-related questions in particular, we shall begin by showing that if we retain all the fields of the SO(8) gauged supergravity, then the maximum degree of supersymmetry that is compatible with any consistent boundary conditions in the ω-deformed SO(8) theory is $\mathcal{N}=3$. A key assumption in reaching this conclusion is that we allow only Dirichlet boundary conditions for the bulk fields with spins $s>1$, since Neumann boundary conditions would give rise to associated propagating spin $s>1$ fields in the dual boundary field theory. We then show that if we truncate the $\mathcal{N}=8$ theory to $\mathcal{N}=6$, the resulting theory still has a non-trivial ω deformation parameter, with $0 \leq \omega \leq \pi/8$, and for two specific inequivalent choices of the ω parameter, namely $\omega=0$ or $\omega=\pi/8$, it is possible to impose boundary conditions that are compatible with the full $\mathcal{N}=6$ supersymmetry. These two theories are related to each by a U(1) electric-magnetic duality.

We construct the full bosonic Lagrangian and supersymmetry transformation of ω-deformed SO(6) gauged supergravity as a consistent truncation of the ω-deformed $\mathcal{N}=8$ theory, generalising similar results for the undeformed SO(6) gauged $\mathcal{N}=6$ supergravity [5]. We compute the three-point correlation functions of the theory at tree-level, focusing on those which depend on the value of the ω parameter. We find that the only such three-point functions involve the coupling of the SO(6) gauge fields with the U(1) gauge field and a scalar or pseudo-scalar field. (The $\mathcal{N}=6$ supergravity has $\mathrm{SO}(6)\times\mathrm{U}(1)$ gauge fields, with the scalars and fermions being charged under SO(6) but not under the U(1).) We also compute these correlation functions in the undeformed $\mathcal{N}=6$ theory. In comparing these results, and finding their possible holographic interpretation, make use of Witten's observation [6] that an electric-magnetic duality rotation in the bulk corresponds to a so called S-transformation of the boundary CFT, in which a global U(1) symmetry is gauged and an off-diagonal Chern-Simons term is introduced. Interestingly, the $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ ABJM model already contains the desired $\mathrm{U}(1)\times\mathrm{U}(1)$ sector. Thus, we suggest that the holographic dual of the $\omega=\pi/8$ theory is not a new CFT, as it would be in Witten's generic framework, but is instead the ABJM model itself, in the sense that the processes involving the Noether current J and those involving the dynamical U(1) in the ABJM model are described by ostensibly distinct bulk theories with $\omega=0$ and $\omega=\pi/8$ respectively. The precise relationship involves the interchange in the CFT correlation functions of a U(1) Noether current and a topological current already present in the ABJM model.

We also give a discussion of the embedding of the ω-deformed theories into higher dimensions. Unlike the ω-deformed $\mathcal{N}=8$ supergravities, whose embedding into eleven dimensions could be expected to involve the introduction of the "dual graviton" in $D=11$ [4], we find that the ω-deformed $\mathcal{N}=6$ gauged supergravities can be embedded into the standard eleven-dimensional or ten-dimensional type IIA supergravities. The essential reason is that the ω-deformed $\mathcal{N}=6$ theories are equivalent, after making an appropriate U(1) duality rotation, at the level of the equations of motion, and since no fields have minimal couplings to the U(1) gauge potential there is no obstruction to performing the necessary dualisation. Furthermore, the consistent Kaluza-Klein sphere reductions always operate at the level of the equations of motion; one cannot write a sphere-reduction ansatz that can be substituted into the higher-dimensional action. For this reason, the embedding of the ω-deformed $\mathcal{N}=6$ theories can be implemented by making the appropriate U(1) duality rotation on the usual embedding ansatz. In the case of an embedding into eleven dimen-

JHEP02(2015)112

sions there would still be a non-local relation between the ansätze for inequivalent values of ω, since the bare U(1) gauge potential appears in the eleven-dimensional metric ansatz. If one instead considers the embedding into ten-dimensional type IIA supergravity, where the U(1) field comes from the Ramond-Ramond 2-form, the bare U(1) gauge potential appears nowhere in the reduction ansatz, and so the embeddings for different values of ω can be locally related.

The plan of this paper is as follows. In section 2 we begin by reviewing some of the key features of the ω-deformed $\mathcal{N} = 8$ gauged supergravities. We then construct an expansion, up to the first few orders in fields, around the maximally-symmetric $\mathcal{N} = 8$ AdS_4 vacuum, with the object of identifying the leading-order interaction terms in which the effect of the ω parameter becomes apparent. We find that this occurs in the trilinear couplings between an SO(6) gauge field, a U(1) gauge field, and a scalar or pseudoscalar field. We also set up the Fefferman-Graham (FG) expansions for all the fields in the AdS_4 background. In section 3, we present in detail the consistent reduction of the ω-deformed $\mathcal{N} = 8$ gauged supergravities to $\mathcal{N} = 6$. We also show that the ω parameter remains non-trivial, in the sense that it parameterises theories related by a U(1) duality rotation that lies outside the $SO^*(12)$ global symmetry group of the theory. In section 4 we study the supersymmetry transformations of the boundary fields in the FG expansion around AdS_4, and we show that the full $\mathcal{N} = 6$ supersymmetry is preserved not only in the undeformed $\omega = 0$ theory but also in the $\omega = \pi/8$ theory. Section 5 contains detailed calculations of the 3-point amplitudes associated with the ω-dependent trilinear couplings identified in section 2. These calculations would also have wider applicability in other situations where one has gauge fields obeying Neumann boundary conditions as as well as gauge fields obeying Dirichlet boundary conditions. We discuss the interpretation of these results in the holographic dual boundary theory in section 6. The paper ends with conclusions in section 7. In appendix A we discuss the embedding of the ω-deformed $\mathcal{N} = 6$ theories in eleven and ten dimensions, and in appendix B we discuss supersymmetric boundary conditions for the boundary fields in the ω-deformed $\mathcal{N} = 8$ gauged supergravities.

2 ω-deformed $\mathcal{N} = 8$ gauged supergravity

In this section we begin by reviewing some of the key aspects of the construction of the ω-deformed $\mathcal{N} = 8$ gauged supergravities. The, for later convenience, we present the first few terms in an expansion of the Lagrangian order-by-order in powers of the fields. Finally in this section, we study the details of the Fefferman-Graham expansions for the linearised solutions around the $\mathcal{N} = 8$ supersymmetric AdS_4 background of the ω-deformed theory.

2.1 Review of the ω-deformed theory

The $\mathcal{N} = 8$ supergravity multiplet consists of the fields

$$(e_\mu^a, \psi_\mu^i, A_\mu^{IJ}, \chi^{ijk}, \phi^{ijk\ell}) , \tag{2.1}$$

where e_μ^a is the vielbein, ψ_μ^i are Weyl gravitini ($i = 1, \ldots, 8$), $A_\mu^{IJ} = A_\mu^{[IJ]}$ are the vector fields ($I, J = 1, \ldots, 8$), $\chi^{ijk} = \chi^{[ijk]}$ are the spin 1/2 Weyl fermions and

$$\phi^{ijk\ell} = \phi^{[ijk\ell]} = (\phi_{ijk\ell})^\star = \frac{1}{4!}\epsilon^{ijk\ell mnpq}\phi_{mnpq} \tag{2.2}$$

are the scalar fields, which parameterize the coset $E_{7(7)}/\mathrm{SU}(8)$. In the 56-dimensional representation, an element of $E_{7(7)}$ can be written as

$$\mathcal{V} = \begin{pmatrix} u_{ij}{}^{IJ} & v_{ijKL} \\ v^{k\ell IJ} & u^{k\ell}{}_{KL} \end{pmatrix}, \tag{2.3}$$

where $u^{ij}{}_{IJ} = (u_{ij}{}^{IJ})^\star$ and $v_{ijIJ} = (v^{ijIJ})^\star$ and

$$\mathcal{V}^\star = \theta\mathcal{V}\theta\,, \qquad \mathcal{V}^\dagger\Omega\mathcal{V} = \Omega\,, \qquad \theta = \begin{pmatrix} 0 & \mathbb{1}_{28} \\ \mathbb{1}_{28} & 0 \end{pmatrix}, \qquad \Omega = \begin{pmatrix} \mathbb{1}_{28} & 0 \\ 0 & -\mathbb{1}_{28} \end{pmatrix}. \tag{2.4}$$

The 56-bein $\mathcal{V}$ transforms by right-multiplication under a rigid $E_{7(7)}$ and by left-multiplication under a local SU(8). Thus the indices $[ij]$ and $[k\ell]$ are local SU(8) indices, whilst $[IJ]$ and $[KL]$ are rigid $E_{7(7)}$ indices. The standard SO(8) gauged $\mathcal{N} = 8$ supergravity theory uses the 56-bein defined above. To obtain the ω-deformed version the theory, it suffices to perform the scalings [4]

$$u_{ij}{}^{IJ} \to e^{i\omega}u_{ij}{}^{IJ}\,, \qquad v_{ijIJ} \to e^{-i\omega}v_{ijIJ}\,, \tag{2.5}$$

with $\omega \in (0, \pi/8]$. The bosonic sector of the resulting ω-deformed theory is described by the Lagrangian

$$\begin{aligned}\mathcal{L}_{\text{bos}} &= \mathcal{L}_{\text{EH}} + \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{scalar}} - eV \\ &= \frac{1}{2}\,e\,R - \frac{1}{8}\,e\left[F^+_{\mu\nu\,IJ}\left(2S^{IJ,KL} - \delta^{IJ}\delta^{KL}\right)F^{+\mu\nu}{}_{KL} + \text{h.c.}\right] - \frac{1}{96}\,e\,P^{ijk\ell}_\mu\,P^\mu_{ijk\ell} \\ &\quad -e\,g^2\left(-\frac{3}{4}\,A_{ij}\,A^{ij} + \frac{1}{24}\,A_i{}^{jk\ell}\,A^i{}_{jk\ell}\right),\end{aligned} \tag{2.6}$$

where $\mathcal{L}_{\text{gauge}}$ and $\mathcal{L}_{\text{scalar}}$ are the kinetic terms for the gauge and scalar fields, V is the potential, and $S^{IJ,KL}$ is a function of the scalar fields defined by

$$\left(u^{ij}{}_{IJ} + v^{ijIJ}\right)S^{IJ,KL} = u^{ij}{}_{KL}\,. \tag{2.7}$$

It can be shown that $S^{IJ,KL} = S^{KL,IJ}$. Further definitions are

$$\begin{aligned}F_{\mu\nu}{}^{IJ} &= 2\,\partial_{[\mu}A_{\nu]}{}^{IJ} - 2\,g\,A_{[\mu}{}^{IM}A_{\nu]}{}^{MJ}\,, \\ P^{ijk\ell}_\mu &= -2\sqrt{2}\left[u^{ij}{}_{IJ}\,D_\mu(A)v^{k\ell IJ} - v^{ijIJ}\,D_\mu(A)u^{k\ell}{}_{IJ}\right], \\ D_\mu(A)u_{ij}{}^{IJ} &= \partial_\mu u_{ij}{}^{IJ} - 2\,g\,A_\mu{}^{M[I}\,u_{ij}{}^{J]M}\,.\end{aligned} \tag{2.8}$$

The SU(8) tensors built out of scalar fields are defined as

$$A^{ij} = \frac{4}{21}\, T_k{}^{ikj} \quad , \qquad A_i{}^{jk\ell} = -\,\frac{4}{3}\, T_i{}^{[jk\ell]}\,, \tag{2.9}$$

where

$$T_i^{jkl} = \Big(e^{-i\omega}\, u^{kl}{}_{IJ} + e^{i\omega}\, v^{klIJ}\big)\, (u_{im}{}^{JK}\, u^{jm}{}_{KI} - v_{imJK}\, v^{jmKI}\Big)\,, \tag{2.10}$$

The local supersymmetry transformations, neglecting terms of cubic or higher order in the fermions, are given by [9]

$$\begin{aligned}
\delta e_\mu{}^a &= \bar\epsilon^i\gamma^a\psi_{\mu i} + \text{h.c.}\,,\\
\delta\psi_\mu^i &= 2D_\mu\epsilon^i + \frac{1}{2\sqrt{2}}H^{-}_{\rho\sigma}{}^{ij}\gamma^{\rho\sigma}\gamma_\mu\epsilon_j + \sqrt{2}gA^{ij}\gamma_\mu\epsilon_j\,,\\
\delta A_\mu{}^{IJ} &= -\left(e^{i\omega}u_{ij}{}^{IJ} + e^{-i\omega}v_{ijIJ}\right)\Big(\bar\epsilon_k\gamma_\mu\chi^{ijk} + 2\sqrt{2}\bar\epsilon^i\psi_\mu{}^j\Big) + \text{h.c.}\,,\\
\delta\chi^{ijk} &= -P_\mu{}^{ijk\ell}\gamma^\mu\epsilon_\ell + \frac{3}{2}\gamma^{\mu\nu}H^{-}_{\mu\nu}{}^{[ij}\epsilon^{k]} - 2gA_\ell{}^{ijk}\epsilon^\ell\,,\\
\left(\delta\mathcal{V}_M{}^{ij}\right)\mathcal{V}^{Mk\ell} &= 2\sqrt{2}\left(\bar\epsilon^{[i}\chi^{jk\ell]} + \frac{1}{24}\varepsilon^{ijk\ell mnpq}\,\bar\epsilon_m\chi_{npq}\right)\,,
\end{aligned} \tag{2.11}$$

where $\mathcal{V}_M{}^{k\ell} = \left(u^{k\ell}{}_{IJ}, v^{k\ell IJ}\right)$, and

$$H^{-}_{\mu\nu}{}^{ij} = \left(e^{-i\omega}u^{ij}{}_{IJ}F_{1\mu\nu}{}^{IJ} + e^{i\omega}v^{ijIJ}F^{-}_{2\mu\nu}{}^{IJ}\right)\,, \tag{2.12}$$

where

$$F^{+}_{1\mu\nu IJ} = \frac{1}{2}\left(\mathrm{i}\,G^{+}_{\mu\nu IJ} + F^{+}_{\mu\nu IJ}\right)\,, \qquad F^{+}_{2\mu\nu}{}^{IJ} = \frac{1}{2}\left(\mathrm{i}\,G^{+}_{\mu\nu IJ} - F^{+}_{\mu\nu IJ}\right)\,, \tag{2.13}$$

$$G^{+\mu\nu}{}_{IJ} = \frac{4\mathrm{i}}{e}\frac{\delta\mathcal{L}_{\text{gauge}}}{\delta F^{+}_{\mu\nu IJ}}\,. \tag{2.14}$$

The covariant derivative of ϵ^i is given by

$$\begin{aligned}
D_\mu\epsilon^i &= \partial_\mu\epsilon^i + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\epsilon^i + \frac{1}{2}Q_\mu{}^i{}_j\epsilon^j\,,\\
Q_\mu{}^i{}_j &= \frac{2}{3}\left[u^{ik}{}_{IJ}D_\mu(A)u_{jk}{}^{IJ} - v^{ikIJ}D_\mu(A)v_{jkIJ}\right]\,.
\end{aligned} \tag{2.15}$$

It is useful to note that the field equations for the vector fields, up to fermionic terms which can be handled straightforwardly [4], and the Bianchi identities for their field strengths, can together be expressed as the 56-dimensional vector equation

$$\partial_\mu\left[e\begin{pmatrix}F_1^{+\mu\nu}\\ F_2^{+\mu\nu}\end{pmatrix} + e\theta\begin{pmatrix}F_1^{+\mu\nu}\\ F_2^{+\mu\nu}\end{pmatrix}^{\star}\right] = 0\,, \tag{2.16}$$

where θ is defined in (2.4). In the limit of vanishing SO(8) coupling g [7], this equation is invariant under a rigid $E_{7(7)}$ symmetry. This equation serves to define $H+_{\mu\nu}$ as well as $G^{+}_{\mu\nu}$, the latter taking the form

$$\mathrm{i}\;(e^{-\,i\omega}u^{ij}{}_{IJ} + e^{\,i\omega}\,v^{ijIJ})\,G^{+}_{\mu\nu\,IJ} = (e^{-\,i\omega}u^{ij}{}_{IJ} - e^{\,i\omega}\,v^{ijIJ})\,F^{+}_{\mu\nu\,IJ}\,. \tag{2.17}$$

The ostensible Sp(56) symmetry of (2.16) for $g = 0 = \omega$ is broken down to $E_7(7)$ by the requirement of the consistency of the transformations that rotate $F_{\mu\nu}$ and $G_{\mu\nu}$ into each other with the equation (2.14). This is a stringent condition, since (2.14) shows that $G_{\mu\nu}$ is not an independent field but rather a functional of $F_{\mu\nu}$ and the scalar fields. The fact that consistency is achieved by for the $E_{7(7)}$ symmetry is seen manifestly from the observation that the following relation holds:

$$\mathcal{V}\begin{pmatrix} F^+_{1\mu\nu} \\ F^+_{2\mu\nu} \end{pmatrix} = \begin{pmatrix} H^+_{\mu\nu} \\ 0 \end{pmatrix} . \tag{2.18}$$

This equation is manifestly $E_{7(7)}$ invariant for vanishing SO(8) coupling constant, and vanishing ω. The introduction of ω-dependent phase factors takes $\mathcal{V}$ outside $E_{7(7)}$ but it is still inside Sp(56), in such a way that the theory is still locally supersymmetric. The further turning on of the ω parameter is consistent with the local SO(8) and with supersymmetry.

In the following section, where we shall consider the consistent truncation of the $\mathcal{N} = 8$ theory, we shall need the identities

$$0 = \left(u^{k\ell}{}_{IJ} + v^{k\ell IJ}\right)\left(u_{ij}{}^{JK}\, u^{mn}{}_{KI} - v_{ijJK}\, v^{mnKI}\right) - \frac{2}{3}\delta^{[m}_{[i} T_{j]}{}^{n]k\ell} , \tag{2.19}$$

$$0 = \frac{4}{3} T_i{}^{jk\ell} + A_{2i}{}^{jk\ell} + 2A_1^{j[k}\delta_i^{\ell]} , \tag{2.20}$$

$$0 = 18 A_1^{ik} A_{1kj} - A^i_{2k\ell m} A_{2j}{}^{k\ell m} - \text{trace} . \tag{2.21}$$

We shall also need the relation [8]

$$Q^{[k}_{\mu\ [i}\delta^{\ell]}{}_{j]} = u_{ij}{}^{IJ} D_\mu(A) u^{k\ell}{}_{IJ} - v_{ijIJ} D_\mu(A) v^{k\ell IJ} . \tag{2.22}$$

Finally, we record a convenient parametrization of the $E_{7(7)}/\mathrm{SU}(8)$ coset element in the so-called symmetric gauge, in which it takes the form

$$\mathcal{V} = \exp\begin{bmatrix} 0 & -\frac{1}{2\sqrt{2}}\phi_{ijkl} \\ -\frac{1}{2\sqrt{2}}\phi^{ijkl} & 0 \end{bmatrix} . \tag{2.23}$$

In this gauge the I, J indices are no longer distinguishable from the i, j indices.

2.2 Expansion of the ω-deformed $\mathcal{N} = 8$ supergravity around maximally supersymmetric AdS$_4$

In what follows, we will use the following abbreviations

$$(\phi \cdot \bar\phi)^{ij}{}_{kl} \equiv \phi^{ijmn}\,\phi_{mnkl} \quad , \qquad (\phi \cdot \bar\phi \cdot \phi)^{ij,\,kl} \equiv \phi^{ijmn}\,\phi_{mnpq}\,\phi^{pqkl}, \qquad a = -\frac{1}{2\sqrt{2}} . \tag{2.24}$$

Using the coset representative in symmetric gauge as given in (2.23), u and v up to fourth order we expand

$$u^{ij}{}_{IJ} = \delta^{ij}{}_{IJ} + \frac{a^2}{2}\,(\phi\cdot\bar\phi)^{ij}{}_{IJ} + \frac{a^4}{4!}\,(\phi\cdot\bar\phi\cdot\phi\cdot\bar\phi)^{ij}{}_{IJ} + \mathcal{O}(\phi^6) ,$$

$$v^{ijIJ} = a\,\phi^{ijIJ} + \frac{a^3}{3!}\,(\phi\cdot\bar\phi\cdot\phi)^{ij,\,IJ} + \mathcal{O}(\phi^5)\,. \tag{2.25}$$

The expansion of $P_\mu^{ijk\ell}$ up to cubic order in fields is

$$\begin{aligned} P_\mu^{ijkl} &= \partial_\mu\phi^{ijkl} + 4\,g\,\phi^{I[ijk}\,A_\mu{}^{l]J}\,\delta_{IJ} \\ &\quad + \frac{a^2}{3}[(\phi\cdot\bar\phi)^{[ij}{}_{IJ}\,\partial_\mu\phi^{kl]IJ} - \phi^{ijMN}\,\phi^{klPQ}\,\partial_\mu\phi_{MNPQ}]\,. \end{aligned} \tag{2.26}$$

In order to derive the gauge kinetic terms we have to solve order by order in $\mathcal{G}^+_{\mu\nu\,IJ}$ equation (2.14). The result is the following

$$\begin{aligned} i\,G^+_{\mu\nu\,IJ} &= F^{+(1)}_{\mu\nu\,IJ} + F^{+(2)}_{\mu\nu\,IJ} - 2\,a\,e^{2i\omega}\phi^{IJKL}\,F^{+(1)}_{\mu\nu\,KL} \\ &\quad -2\,a\,e^{2i\omega}\,\phi^{IJKL}\,F^{+(2)}_{\mu\nu\,KL} + 2\,a^2\,e^{2i\omega}\,\phi^{IJMN}\,\phi^{MNKL}F^{+(1)}_{\mu\nu\,KL} + \cdots \end{aligned} \tag{2.27}$$

where we have denoted with $F^{(1)}$ and $F^{(2)}$ the abelian and nonabelian part of the field strengths respectively

$$F^{(1)IJ}_{\mu\nu} = 2\,\partial_{[\mu}A_{\nu]}{}^{IJ}\,, \qquad F^{(2)IJ}_{\mu\nu} = -2\,g\,A_{[\mu}{}^{IM}A_{\nu]}{}^{MJ}\,. \tag{2.28}$$

Using the above lemmata, we find

$$\begin{aligned} A^{ij} &= +e^{-\mathrm{i}\,\omega}\,\delta^{ij} + e^{-\mathrm{i}\,\omega}\,\delta^{ij}\,\frac{a^2}{24}\,|\phi|^2 + e^{\mathrm{i}\,\omega}\,\frac{a^3}{3}\,(\phi\cdot\bar\phi\cdot\phi)^{iI,\,jJ}\,\delta_{IJ} \\ &\quad + e^{-\mathrm{i}\,\omega}\left[\delta^{ij}\,\frac{a^4}{864}\,(|\phi|^2)^2 - \frac{a^4}{6}\,(\phi\cdot\bar\phi)^{Ki}{}_{LI}\,(\phi\cdot\bar\phi)^{Lj}{}_{KJ}\,\delta^{IJ}\right] \\ &\quad + e^{\mathrm{i}\,\omega}\left[-\frac{19a^5}{420}(\phi\cdot\bar\phi\cdot\phi\cdot\bar\phi\cdot\phi)^{iI,jI} + \frac{a^5}{63}|\phi|^2(\phi\cdot\bar\phi\cdot\phi)^{iI,jI}\right. \\ &\qquad \left. + \frac{a^5}{21}(\phi\cdot\bar\phi)^{i\ell}{}_{KI}(\phi\cdot\bar\phi)^{JK}{}_{k\ell}\phi^{kjIJ}\right]. \end{aligned} \tag{2.29}$$

$$\begin{aligned} A_i{}^{jkl} &= -e^{\mathrm{i}\,\omega}\,2\,a\,\delta_{iI}\,\phi^{Ijkl} - e^{-\mathrm{i}\,\omega}\,3\,a^2\,(\phi\cdot\bar\phi)^{[jk}{}_{iI}\,\delta^{l]I} \\ &\quad - e^{\mathrm{i}\,\omega}\left[\frac{a^3}{9}\,\delta_{iI}\,\phi^{Ijkl}\,|\phi|^2 + 2\,a^3\,\phi^{KI[jk}\,\phi_{Kimn}\,\phi^{l]Jmn}\,\delta^{IJ}\right] \\ &\quad + e^{-\mathrm{i}\,\omega}\left[\frac{a^4}{12}(\phi\cdot\bar\phi\cdot\phi\cdot\bar\phi)^{[jk}{}_{iI}\delta^{\ell]}_I - \frac{7a^4}{72}|\phi|^2(\phi\cdot\bar\phi)^{[jk}{}_{iI}\delta^{\ell]}_I\right. \\ &\qquad \left. + \frac{a^4}{3}(\phi\cdot\bar\phi)^{m[j}{}_{nI}\delta^k_I(\phi\cdot\bar\phi)^{\ell]n}{}_{im} - a^4(\phi\cdot\bar\phi)^{[jk}{}_{IJ}(\phi\cdot\bar\phi)^{\ell]I}{}_{iJ}\right]. \end{aligned} \tag{2.30}$$

In obtaining the above results, we have used the following properties, related to self-duality of the scalar fields

$$\phi^{rstm}\,\phi_{rsti} = +\frac{1}{8}\,\delta_i{}^m\,|\phi|^2\,,$$

JHEP02(2015)112

$$\phi^{rmnp}\,\phi_{rijk} \;=\; -\frac{1}{16}\,\delta_{ijk}{}^{mnp}\,|\phi|^2 + \frac{9}{4}\,\delta_{[i}{}^{[m}\,(\phi\cdot\bar\phi)^{np]}{}_{jk]}\,. \tag{2.31}$$

Note that in (2.29) and (2.30) the even and odd powers of scalar fields are multiplied by different ω phases. When computing the scalar potential we take the T tensor components times their complex conjugates and the ω dependence drops out in the terms which contain an even number of scalar fields. An ω dependent phase shows up in the other terms, containing an odd number of scalar fields. This is consistent with the fact that the value of the cosmological constant and the scalar spectrum (obtained from the order zero and order two terms) are insensitive to the ω-deformation. Finally notice that, if we want to compute the scalar potential up to fourth order in the scalar fields we only need to compute $A_i{}^{jkl}$ up to third order, due to the absence of the zeroth order contribution to this T-tensor component.

Next, we expand the bosonic Lagrangian (2.6) to fourth order in excitations around the maximally supersymmetric AdS_4 vacuum. The vacuum solution corresponds to $\phi_{ijkl} = 0 = A_\mu{}^{IJ}$. Using the formula given above, we find

$$\begin{aligned}
e^{-1}\,\mathcal{L}_{\rm scalar} \;=\; &-\frac{1}{96}\,\partial_\mu\phi^{ijkl}\,\partial^\mu\phi_{ijkl} + \frac{1}{24}\,g\,(\partial_\mu\phi^{ijkl})\,\phi_{Iijk}\delta_{lJ}\,A^{\mu\,IJ} + \text{c.c.}\\
&-\frac{a^2}{144}\,(\partial_\mu\phi^{ijmn})(\partial^\mu\phi_{mnkl})\,(\phi\cdot\bar\phi)^{ij}{}_{kl}\\
&-\frac{a^2}{286}\left[\phi^{ijmn}\,\phi^{klpq}\,(\partial_\mu\phi_{ijkl})(\partial^\mu\phi_{mnpq}) + \text{c.c}\right]\\
&-\frac{1}{24}\,g^2\left[\frac{1}{8}\,|\phi|^2\,A_\mu{}^{IJ}\,A^{\mu\,IJ} + 3\,(\phi\cdot\bar\phi)^{IK}{}_{JL}\,A_\mu{}^{IJ}\,A^{\mu\,KL}\right].
\end{aligned}\tag{2.32}$$

The expansion of the gauge kinetic terms, on the other hand, up to fourth order in field, yields

$$\begin{aligned}
e^{-1}\mathcal{L}_{\rm gauge} \;=\; &-\frac{1}{2}\,\partial_{[\mu}A_{\nu]}{}^{IJ}\,\partial^\mu A^{\nu\,IJ} + g\,\partial_{[\mu}A_{\nu]}{}^{IJ}\,A^{\mu\,IM}A^{\nu\,MJ}\\
&+\mathrm{Re}\{a e^{2i\omega}\phi^{IJKL}\}\,\partial_{[\mu}A_{\nu]}{}^{IJ}\,\partial^\mu A^{\nu\,KL}\\
&+\frac{1}{2}\,\mathrm{Im}\{a e^{2i\omega}\phi^{IJKL}\}\,\epsilon^{\mu\nu\rho\sigma}\partial_\mu A_\nu{}^{IJ}\,\partial_\rho A_\sigma{}^{KL}\; +\\
&-\mathrm{Re}\{a^2 e^{2i\omega}(\phi\cdot\phi)^{IJ,\,KL}\}\,\partial_{[\mu}A_{\nu]}{}^{IJ}\,\partial^\mu A^{\nu\,KL}\\
&-\frac{1}{2}\,\mathrm{Im}\{a^2 e^{2i\omega}(\phi\cdot\phi)^{IJ,\,KL}\}\,\epsilon^{\mu\nu\rho\sigma}\,\partial_\mu A_\nu{}^{IJ}\,\partial_\rho A_\sigma{}^{KL}\\
&-2\,g\,\mathrm{Re}\{a\,e^{2i\omega}\,\phi^{IJKL}\}\,\partial_{[\mu}A_{\nu]}{}^{IJ}A^{\mu\,KM}A^{\nu\,ML}\\
&-g\,\mathrm{Im}\{a\,e^{2i\omega}\,\phi^{IJKL}\}\,\epsilon^{\mu\nu\rho\sigma}\,\partial_\mu A_\nu{}^{IJ}A_\rho{}^{KM}A_\sigma{}^{ML}\\
&-\frac{1}{2}\,g^2\,A_{[\mu}{}^{IM}A_{\nu]}{}^{MJ}A^{\mu\,IN}A^{\nu\,NJ}\,.
\end{aligned}\tag{2.33}$$

Finally, the potential up to fifth order in scalar fields, is given by

$$V \;=\; -6 - \frac{a^2}{3}\,|\phi|^2 + \frac{a^4}{216}\,(|\phi|^2)^2 - \frac{a^4}{3}\,(\phi\cdot\bar\phi)^{Ki}{}_{LI}\,(\phi\cdot\bar\phi)^{Li}{}_{KI}$$

$$-\left[\frac{a^5}{10}e^{\mathrm{i}\,2\omega}(\phi\cdot\bar{\phi}\cdot\phi\cdot\bar{\phi}\cdot\phi)^{iI,iI}-\frac{a^5}{36}e^{\mathrm{i}\,2\omega}|\phi|^2(\phi\cdot\bar{\phi}\cdot\phi)^{iI,iI}+c.c\right]. \tag{2.34}$$

Note that the ω dependence of this $\mathcal{N}=8$ potential arises first at the fifth order in powers of the scalar field. By contrast, as discussed in section 3, the ω parameter does not enter at all in the scalar potential V of the $\mathcal{N}=6$ theory.

2.3 Fefferman-Graham expansions in the linearized ω-deformed $\mathcal{N}=8$ supergravity

Here we study the variation of the boundary fields arising in the Fefferman-Graham (FG) expansion, in which certain convenient gauge choices are made for the fields of spins $s\geq 1$. Since we shall be using the symmetric gauge for the representative of the the coset $E_{7(7)}/\mathrm{SU}(8)$, there will be no distinction between SU(8) and SO(8) indices. Therefore, in this subsection we shall use $I,J,K,\ldots$ to denote SO(8) indices, which then allows us to use the indices $i,j,k,\ldots$ to label the coordinates of the three-dimensional boundary of AdS$_4$. For the fermionic fields, the avoidance of SU(8) indices will be facilitated by going over to Majorana basis. In doing so, we shall, for convenience, include ω-dependent phases as follows:

$$e^{\frac{i}{2}\omega}\epsilon_L+e^{-\frac{i}{2}\omega}\epsilon_R\ \ \to \epsilon^I\ \ (\text{Majorana})\,,\qquad e^{-\frac{i}{2}\omega}\chi_R+e^{\frac{i}{2}\omega}\chi_L\ \ \to\ \ \chi^{IJK}\ \ (\text{Majorana})\,, \tag{2.35}$$

where the SU(8) indices on the chiral spinors are suppressed. It will also prove to be convenient to define the real and imaginary parts of the scalar fields as

$$\phi^{IJKL}=\mathcal{S}^{IJKL}+\mathrm{i}\,\mathcal{P}^{IJKL}\,. \tag{2.36}$$

It follows from (2.2) that

$$\mathcal{S}^{IJKL}=\frac{1}{4!}\,\epsilon^{IJKLMNPQ}\,\mathcal{S}_{MNPQ}\,,\qquad \mathcal{P}^{IJKL}=-\frac{1}{4!}\,\epsilon^{IJKLMNPQ}\,\mathcal{P}_{MNPQ}\,. \tag{2.37}$$

Choosing the gauges

$$e_0^{\hat{0}}=\frac{1}{z_0}\,,\qquad e_i^{\hat{0}}=0\,,\qquad e_0^{\hat{r}}=0\,,\qquad A_0^{IJ}=0\,,\qquad \psi_0^I=0\,, \tag{2.38}$$

the equations of motion then determine the falloff behaviour of the fields to be

$$\begin{aligned}
e_i^{\hat{r}} &= \frac{1}{z_0}(e^{\hat{r}}_{(0)i}+z_0^2 i\,e^{\hat{r}}_{(2)_i}+z_0^3\,e^{\hat{r}}_{(3)i}+\cdots)\,,\\
A_i^{IJ} &= A^{IJ}_{(0)i}+z_0\,A^{IJ}_{(1)i}+\cdots\,,\\
\mathcal{S}^{IJKL} &= z_0\,\mathcal{S}^{IJKL}_{(1)}+z_0^2\,\mathcal{S}^{IJKL}_{(2)}+\cdots\,,\\
\mathcal{P}^{IJKL} &= z_0\,\mathcal{P}^{IJKL}_{(1)}+z_0^2\,\mathcal{P}^{IJKL}_{(2)}+\cdots\,,\\
\psi_i^I &= z_0^{-\frac{1}{2}}\psi^I_{(0)i+}+z_0^{\frac{1}{2}}\psi^I_{(2)i-}+z_0^{\frac{3}{2}}\psi^I_{(3)i}+\cdots\,,\\
\chi^{IJK} &= z_0^{\frac{3}{2}}\chi_+^{IJK}+z_0^{\frac{3}{2}}\chi_-^{IJK}+\cdots\,,
\end{aligned} \tag{2.39}$$

JHEP02(2015)112

The asymptotic Killing spinor can be expressed as

$$\epsilon^I = z_0^{-\frac{1}{2}}\epsilon_+^I + z_0^{\frac{1}{2}}\epsilon_-^I + z_0^{\frac{3}{2}}\epsilon_{(3)}^I + \cdots\,. \tag{2.40}$$

Plugging the FG expansions of various fields into the supersymmetry transformation rules, we can extract the supersymmetry variations of the coefficients in the expansions. Firstly, we see that

$$\begin{aligned}\delta e^{\hat{r}}_{(0)i} &= \bar{\epsilon}^I_+ \gamma^{\hat{r}}_{(0)} \psi^I_{(0)i+}\,,\\ \delta\psi^I_{(0)i+} &= \frac{1}{2}\mathcal{K}^{ab}_{(0)i}\gamma_{ab}\epsilon^I_+ + \sqrt{2}A^{IJ}_{(0)i}\epsilon^J_+\,,\end{aligned} \tag{2.41}$$

where $\mathcal{K}^{ab}_{(0)i}$ is the super-torsion constructed from $\psi^I_{(0)i+}$, and we have used the fact that $\partial_i\epsilon_+ = \frac{1}{2}\gamma_i\epsilon_-$. (See, for example, [33] for the explicit solution for the Killing spinors in AdS_{d+1} in Poincaré coordinates.) The supersymmetry variation of the boundary data of the spin-1 field is given as

$$\begin{aligned}\delta A^{IJ}_{(0)i} &= -\Big(\cos 2\omega\ \epsilon^K_+\gamma_{(0)i}\chi^{IJK}_+ + \mathrm{i}\sin 2\omega\ \epsilon^K_+\gamma_{(0)i}\gamma_5\chi^{IJK}_-\Big) + \cdots\,,\\ \delta A^{IJ}_{(1)i} &= \Big[-\mathcal{S}^{IJKL}_{(1)}\epsilon^M_+\gamma_{(0)i}\chi^{KLM}_+ - \mathrm{i}\mathcal{P}^{IJKL}_{(1)}\epsilon^M_+\gamma_{(0)i}\gamma_5\chi^{KLM}_- - 2\sqrt{2}\epsilon^{[I}_+\psi^{J]}_{i(3)-}\\ &\qquad + D_i(A_{(0)})\left(\cos 2\omega\ \epsilon^K_+\chi^{IJK}_- + \mathrm{i}\sin 2\omega\ \epsilon^K_+\gamma_5\chi^{IJK}_+\right)\Big] + \cdots\,.\end{aligned} \tag{2.42}$$

where the ellipses refer to term depending on $\psi^r_{(0)i+}$, which vanish for the Dirichlet boundary conditions that we shall impose on the gravitini in the next section when we analyze the supersymmetry-preserving boundary conditions. There remains the supersymmetry variation of the boundary data of the spin-$\frac{1}{2}$ and spin-0 fields, which take the form

$$\begin{aligned}\delta\chi^{IJK}_+ &= -\mathcal{S}^{IJKL}_{(2)}\epsilon^L_+ + 2\mathrm{i}\mathcal{P}^{IJKL}_{(1)}\gamma_5\epsilon^L_- - \mathrm{i}\not{D}\mathcal{P}^{IJKL}_{(1)}\gamma_5\epsilon^L_+\,,\\ \delta\chi^{IJK}_- &= 2\mathcal{S}^{IJKL}_{(1)}\epsilon^L_- - \mathrm{i}\mathcal{P}^{IJKL}_{(2)}\gamma_5\epsilon^L_+ + \not{D}\mathcal{S}^{IJKL}_{(1)}\epsilon^L_+\,,\\ \delta\mathcal{S}^{IJKL}_{(1)} &= 4\left(\bar{\epsilon}^{[I}_+\chi^{JKL]}_- + \frac{1}{4!}\varepsilon^{IJKLMNPQ}\bar{\epsilon}^M_+\chi^{NPQ}_-\right),\\ \delta\mathcal{P}^{IJKL}_{(1)} &= -4\mathrm{i}\left(\bar{\epsilon}^{[I}_+\gamma_5\chi^{JKL]}_+ - \frac{1}{4!}\varepsilon^{IJKLMNPQ}\bar{\epsilon}^M_+\gamma_5\chi^{NPQ}_+\right),\\ \delta\mathcal{S}^{IJKL}_{(2)} &= 4\Bigg(\bar{\epsilon}^{[I}_-\chi^{JKL]}_+ + \frac{1}{4!}\varepsilon^{IJKLMNPQ}\bar{\epsilon}^M_-\chi^{NPQ}_+\\ &\qquad + \bar{\epsilon}^{[I}_+\not{D}\chi^{JKL]}_+ + \frac{1}{4!}\varepsilon^{IJKLMNPQ}\bar{\epsilon}^M_+\not{D}\chi^{NPQ}_+\Bigg),\\ \delta\mathcal{P}^{IJKL}_{(2)} &= -4\mathrm{i}\Bigg(\bar{\epsilon}^{[I}_-\gamma_5\chi^{JKL]}_- - \frac{1}{4!}\varepsilon^{IJKLMNPQ}\bar{\epsilon}^M_-\gamma_5\chi^{NPQ}_-\\ &\qquad - \bar{\epsilon}^{[I}_+\gamma_5\not{D}\chi^{JKL]}_- + \frac{1}{4!}\varepsilon^{IJKLMNPQ}\bar{\epsilon}^M_+\gamma_5\not{D}\chi^{NPQ}_-\Bigg).\end{aligned} \tag{2.43}$$

It should be noted that several compensating transformations have been used when deriving the above results. The reason for this is that the gauge choices adopted in the FG expansion are not preserved under the supersymmetry transformations alone. It is necessary to make certain compensating transformations of the fields using the diffeomorphism, local Lorentz and local SO(8) symmetries, in order to maintain the orignal gauge choices. We denote the corresponding transformation parameters by (ξ^μ, $\Lambda^a{}_b$ and O^{IJ}), respectively. The gauge choice for the vielbein (see (2.38)) is preserved by accompanying the supersymmetry transformations with compensating diffeomorphism and local Lorentz transformations, whose parameters are related to the supersymmetry parameter by

$$\xi^\mu = -\int dr(\bar{\epsilon}^I\gamma^{\hat{0}}\psi^I_{\hat{r}}e^{\mu\hat{r}})\,, \qquad \Lambda^{\hat{0}}{}_i = -\bar{\epsilon}^I\gamma^{\hat{0}}\psi^I_i\,. \tag{2.44}$$

To maintain the gauge choice $A_0^{IJ}=0$ requires a compensating SO(8) transformation with parameters determined by the conditions

$$\partial_{z_0}O^{IJ} = -\delta_\epsilon A_0^{IJ}\,. \tag{2.45}$$

The O^{IJ} can be solved order by order in z_0, with, at leading order,

$$O^{IJ} = z_0\,O^{IJ}_{(1)} + \cdots\,, \qquad O^{IJ}_{(1)} = (\cos 2\omega \;\; \epsilon_{K+}\chi^{IJK}_- + \mathrm{i}\,\sin 2\omega \;\; \epsilon_{K+}\gamma_5\chi^{IJK}_+)\,. \tag{2.46}$$

The compensating transformation O^{IJ} explains the derivative term in the supersymmetry variation of $A^{IJ}_{(1)i}$. To maintain the gauge condition $\psi^r_0 = 0$, the $\epsilon^i_{(3)}$ and higher-order coefficients in the FG expansion of the supersymmetry transformation parameter need to be modified, but these do not affect the result at the order to which we are working. The compensating SU(8) transformation needed for maintaining the symmetric gauge takes the form $\Pi^I{}_J = \frac{2}{3}\left(u^{IM}{}_{KL}\delta_\epsilon u_{JM}{}^{KL} - v^{IMKL}\delta_\epsilon v_{JMKL}\right)$. It can be seen that $\Pi^I{}_J \sim \mathcal{O}(z_0^2)$ after using the FG expansions of the scalar fields. Therefore, $\Pi^I{}_J$ will not contribute to the variation of the leading falloff coefficients.

3 Consistent truncation to ω-deformed SO(6) gauged $\mathcal{N}=6$ supergravity

In this section, we shall construct the full bosonic Lagrangian and supersymmetry transformations of ω-deformed SO(6) gauged supergravity as a consistent truncation of the $\mathcal{N}=8$ theory summarized in the previous section. To keep the notation simple, we shall use the same indices i,j and I,J that in the previous section ran from 1 to 8, but now in this section they will run from 1 to 6 for the $\mathcal{N}=6$ theory. The consistent truncation of the $\mathcal{N}=8$ theory is achieved by the rules

$$A^{7I}=0\,, \qquad A^{8I}=0\,, \qquad \phi^{IJK7}=0\,, \qquad \phi^{IJK8}=0\,, \tag{3.1}$$

in the bosonic sector, and by

$$\psi^7_\mu = \psi^8_\mu = 0\,, \qquad \chi^{IJ7} = \chi^{IJ8} = 0\,, \tag{3.2}$$

JHEP02(2015)112

in the fermionic sector. It is straightforward to check that the supersymmetry transformations of the truncated fields remain vanishing. Introducing the notation

$$\begin{aligned} A_\mu^{78} &:= A_\mu\,, \qquad P_\mu^{ij78} := P_\mu^{ij}\,, \qquad Q^{78} := Q_\mu\,, \\ u^{ij}{}_{78} &:= u^{ij}\,, \qquad u_{78}{}^{IJ} := u^{IJ}\,, \qquad u^{78}{}_{78} := u\,, \end{aligned} \tag{3.3}$$

with self-explanatory similar definitions for (v^{ij}, v^{IJ}, v), the truncation rules (3.1) lead to the following result for the full bosonic Lagrangian of the ω-deformed SO(6) gauged supergravity:

$$\begin{aligned} \mathcal{L}_{\rm bos} = \frac{1}{2}\, e\, R - \frac{1}{8} e \Big[& F^{+\,IJ}_{\mu\nu} \left(2S^{IJ,KL} - \delta^{IK}\delta^{JL}\right) F^{+\mu\nu}_{KL} + 4F^{+}_{\mu\nu} S^{IJ} F^{+\mu\nu}_{IJ} \\ & + 2F^{+}_{\mu\nu}(2S-1)F^{+\mu\nu} + \text{h.c.}\Big] - \frac{1}{96}\, e\, P^{ijk\ell}_{\mu}\, P^{\mu}_{ijk\ell} - \frac{1}{8}\, e\, P^{ij}_{\mu}\, P^{\mu}_{ij} - eV\ , \end{aligned} \tag{3.4}$$

where the S functions are defined by the relations

$$\begin{aligned} \left[(u+v)(u^{ij}{}_{KL}+v^{ijKL})-(u^{ij}+v^{ij})(u_{KL}+v_{KL})\right] S^{KL,IJ} &= (u+v)u^{ij}{}_{IJ}-(u^{ij}+v^{ij})u_{IJ}\,, \\ S^{IJ} &= (u+v)^{-1}\left[u_{IJ}-(u_{KL}+v_{KL})S^{KL,IJ}\right]\,, \\ S &= (u+v)^{-1}\left[u-(u_{IJ}+v^{IJ})S^{IJ}\right]\,, \end{aligned} \tag{3.5}$$

as can be seen from (2.7) and the truncation conditions (3.1). The definitions (2.8) hold for the $\mathcal{N}=6$ theory with all the indices restricted to run from 1 to 6, while P_μ^{ij} is defined as

$$P_\mu^{ij} = -2\sqrt{2}\left[u^{ij}{}_{IJ} D_\mu(A) v^{IJ} - v^{ijIJ} D_\mu(A) u_{IJ}\right]\,. \tag{3.6}$$

The scalar potential for the $\mathcal{N}=6$ theory takes the form

$$V = g^2\left(-A_{1ij}\,A_1^{ij} + \frac{1}{18}\,A_{2i}{}^{jk\ell}\,A_2{}^{i}{}_{jk\ell} + \frac{1}{3}\,A_{2i}{}^{j}\,A_2{}^{i}{}_{j}\right)\,, \tag{3.7}$$

where the functions

$$A_1^{ij} = \frac{4}{15}\,T_k{}^{ikj}\,, \qquad A_{2i}{}^{jk\ell} = -\frac{4}{3}\,T_i{}^{[jk\ell]}\,, \qquad A_{2i}{}^{j} = -\frac{4}{3}\,T_i{}^{j}\,, \tag{3.8}$$

are defined in terms of the tensors

$$\begin{aligned} T_i^{jkl} &= \frac{3}{2}\left(e^{-i\omega}\,u^{kl}{}_{IJ} + e^{i\omega}\,v^{klIJ}\right)\left(u_{im}{}^{JK}\,u^{jm}{}_{KI} - v_{imJK}\,v^{jmKI}\right) - \delta_i^j S^{kl}\,, \\ S^{kl} &= \frac{3}{5}\left(e^{-i\omega}\,u^{kl}{}_{IJ} + e^{i\omega}\,v^{klIJ}\right)\left(u_{ij}{}^{JK}\,u^{ij}{}_{KI} - v_{ijJK}\,v^{ijKI}\right)\,. \end{aligned} \tag{3.9}$$

and

$$\begin{aligned} T_i{}^{j} &= \frac{3}{2}\left(e^{-i\omega}\,u_{IJ} + e^{i\omega}\,v^{IJ}\right)\left(u_{im}{}^{JK}\,u^{jm}{}_{KI} - v_{imJK}\,v^{jmKI}\right) - \delta_i^j S\,, \\ S &= \frac{3}{5}\left(e^{-i\omega}\,u_{IJ} + e^{i\omega}\,v^{IJ}\right)\left(u_{ij}{}^{JK}\,u^{ij}{}_{KI} - v_{ijJK}\,v^{ijKI}\right)\,. \end{aligned} \tag{3.10}$$

JHEP02(2015)112

The result (3.7) is obtained by restricting the free indices of of (2.21) to run from 1 to 6 and then taking the trace. The relations (3.8) follow from (2.20), while (3.9) and (3.10) follow from (2.19) by the restriction of the free indices to lie in the SO(6) and U(1) directions.

The local supersymmetry transformations of the $\mathcal{N} = 6$ theory are obtained, up to cubic fermions, from those of the $\mathcal{N} = 8$ theory by applying the consistent truncation rules (3.1), and they take the form

$$
\begin{aligned}
\delta e_\mu{}^a &= \bar\epsilon^i \gamma^a \psi_{\mu i} + \text{h.c.}\,, \\
\delta\psi_\mu^i &= 2D_\mu \epsilon^i + \frac{1}{2\sqrt{2}} H^{-}_{\rho\sigma}{}^{ij} \gamma^{\rho\sigma}\gamma_\mu \epsilon_j + \sqrt{2} g A_1^{ij} \gamma_\mu \epsilon_j\,, \\
\delta A_\mu{}^{IJ} &= -\Big[\left(e^{i\omega} u_{ij}{}^{IJ} + e^{-i\omega} v_{ijIJ}\right) \left(\bar\epsilon_k \gamma_\mu \chi^{ijk} + 2\sqrt{2}\bar\epsilon^i \psi_\mu{}^j\right) \\
&\qquad\qquad + 2\left(e^{i\omega} u^{IJ} + e^{-i\omega} v_{IJ}\right) \bar\epsilon_k \gamma_\mu \chi^k + \text{h.c.}\Big]\,, \\
\delta A_\mu &= -\left(e^{i\omega} u_{ij} + e^{-i\omega} v_{ij}\right) \left(\bar\epsilon_k \gamma_\mu \chi^{ijk} + 2\sqrt{2}\bar\epsilon^i \psi_\mu{}^j\right) + 2(e^{i\omega} u + e^{-i\omega} v)\bar\epsilon_k \gamma_\mu \chi^k + \text{h.c.}\,, \\
\delta\chi^{ijk} &= -P_\mu{}^{ijk\ell} \gamma^\mu \epsilon_\ell + \frac{3}{2} \gamma^{\mu\nu} H^{-}_{\mu\nu}{}^{[ij} \epsilon^{k]} - 2g A_{2\ell}{}^{ijk} \epsilon^\ell\,, \\
\delta\chi^i &= -P_\mu{}^{ij} \gamma^\mu \epsilon_j + \frac{1}{2} \gamma^{\mu\nu} H^{-}_{\mu\nu} \epsilon^i - 2g A_{2j}{}^i \epsilon^j\,, \\
\left(\delta\mathcal{V}_M{}^{ij}\right) \mathcal{V}^M &= \sqrt{2}\left(\bar\epsilon^{[i}\chi^{j]} + \frac{1}{12} \varepsilon^{ijk\ell mn}\, \bar\epsilon_k \chi_{\ell mn}\right)\,,
\end{aligned}
\tag{3.11}
$$

where $\mathcal{V}_M{}^{k\ell} = \left(u^{k\ell}{}_{IJ}, v^{k\ell IJ}\right)$. We have made the following definitions. Firstly we define the 32-bein

$$
\mathcal{V} = \begin{pmatrix} U & V \\ V^\star & U^\star \end{pmatrix}\,, \qquad U := \begin{pmatrix} u_{ij}{}^{KL} & u_{ij} \\ u^{\ KL} & u \end{pmatrix}\,, \qquad V := \begin{pmatrix} v_{ijKL} & v_{ij} \\ v_{KL} & v \end{pmatrix}\,. \tag{3.12}
$$

Recalling the definitions (3.3), it is straightforward to check that $\mathcal{V}$ satisfies

$$
\mathcal{V}^\star = \theta \mathcal{V} \theta\,, \qquad \mathcal{V}^\dagger \Omega \mathcal{V} = \Omega\,, \tag{3.13}
$$

where

$$
\theta = \begin{pmatrix} 0 & \mathbb{1}_{16} \\ \mathbb{1}_{16} & 0 \end{pmatrix}\,, \qquad \Omega = \begin{pmatrix} \mathbb{1}_{16} & 0 \\ 0 & -\mathbb{1}_{16} \end{pmatrix} \tag{3.14}
$$

The relations (3.13), together with the form for $\mathcal{V}$ in (3.12), show that $\mathcal{V}$ is an element of $SO^\star(12)$.

Next we define

$$
\begin{aligned}
H^{-}_{\mu\nu}{}^{ij} &= \left(e^{-i\omega} u^{ij}{}_{IJ} F_{1\mu\nu}{}^{IJ} + e^{i\omega} v^{ijIJ} F^{-}_{2\mu\nu}{}^{IJ}\right) + \left(e^{-i\omega} u^{ij} F_{1\mu\nu} + e^{i\omega} v^{ij} F^{-}_{2\mu\nu}\right)\,, \\
H^{-}_{\mu\nu} &= \left(e^{-i\omega} u_{IJ} F_{1\mu\nu}{}^{IJ} + e^{i\omega} v^{IJ} F^{-}_{2\mu\nu}{}^{IJ}\right) + \left(e^{-i\omega} u F_{1\mu\nu} + e^{i\omega} v F^{-}_{2\mu\nu}\right)\,,
\end{aligned}
\tag{3.15}
$$

JHEP02(2015)112

where $F^+_{1\mu\nu IJ}$ and $F^+_{2\mu\nu IJ}$ are defined as in (2.13), but with the indices now running from 1 to 6, and

$$F^+_{1\mu\nu} = (\mathrm{i}\, G^+_{\mu\nu} + F^+_{\mu\nu})\,, \qquad F^+_{2\mu\nu} = (\mathrm{i}\, G^+_{\mu\nu} - F^+_{\mu\nu})\,. \tag{3.16}$$

We also have the definition similar to (2.14), but now with the indices running from 1 to 6, namely

$$G^{+\mu\nu} = \frac{4\mathrm{i}}{e}\frac{\delta\mathcal{L}_{\text{gauge}}}{\delta F^+_{\mu\nu}}\,. \tag{3.17}$$

The covariant derivative of ϵ^i is now defined as

$$\begin{aligned}
D_\mu\epsilon^i &= \partial_\mu\epsilon^i + \frac{1}{4}\omega_\mu{}^{ab}\gamma_{ab}\epsilon^i + \frac{1}{2}Q_\mu{}^i{}_j\epsilon^j\,,\\
Q_\mu{}^i{}_j &= -u_{\ell j}{}^{IJ}D_\mu(A)u^{i\ell}{}_{IJ} - v_{\ell jIJ}D_\mu(A)v^{i\ell IJ} - u_{\ell j}\partial_\mu u^{i\ell} + v_{\ell j}\partial_\mu v^{i\ell}\\
&\quad + \frac{1}{10}\delta^i_j\left[u_{k\ell}{}^{IJ}D_\mu(A)u^{k\ell}{}_{IJ} - v_{k\ell IJ}D_\mu(A)v^{k\ell IJ} + u_{k\ell}\partial_\mu u^{k\ell} - v_{k\ell}\partial_\mu v^{k\ell}\right]\,.
\end{aligned} \tag{3.18}$$

In the limit of vanishing SO(6) coupling constant, the field equations are invariant under $SO^\star(12)$ duality transformations, as can be seen from the field equations analogous to (2.16), and the fact that

$$\mathcal{V}\begin{pmatrix} F^+_{1\mu\nu IJ}\\ F^+_{1\mu\nu}\\ F^{+KL}_{2\mu\nu}\\ F^+_{2\mu\nu}\end{pmatrix} = \begin{pmatrix} H^+_{\mu\nu ij}\\ H^+_{\mu\nu}\\ 0\\ 0\end{pmatrix}. \tag{3.19}$$

One can show that $\hat{U}\mathcal{V}$ satisfies the same properties as $\mathcal{V}$, provided that $\hat{U}$ takes the form

$$\hat{U} = \begin{pmatrix} U_{16\times16} & 0\\ 0 & U^*_{16\times16}\end{pmatrix}, \quad U_{16\times16} = \begin{pmatrix} \mathrm{SU}(6)_{15\times15} & 0\\ 0 & 1\end{pmatrix}\times\begin{pmatrix} e^{\mathrm{i}\,\alpha}\mathbb{1}_{15} & 0\\ 0 & e^{-3\mathrm{i}\,\alpha}\end{pmatrix}, \tag{3.20}$$

where the $\mathrm{U}(1)_R$ factor is given by

$$\hat{V} = \begin{pmatrix} V_{16\times16} & 0\\ 0 & V^*_{16\times16}\end{pmatrix}, \quad V = \begin{pmatrix} e^{\mathrm{i}\,\alpha}\mathbb{1}_{15} & 0\\ 0 & e^{-3\mathrm{i}\,\alpha}\end{pmatrix}. \tag{3.21}$$

Therefore $\hat{U}$ is an element of R-symmetry group U(6) and consequently $\hat{U}\mathcal{V}$ also parameterises $SO^*(12)$. Note that since $\hat{V}$ is an element inside the U(6) subgroup of $SO^*(12)$ and $\mathcal{V}$ is an element in the coset $SO^*(12)/\mathrm{U}(6)$, by definition

$$\hat{V}\mathcal{V}(\phi) = \mathcal{V}(\phi')\hat{V}, \quad \phi'^{IJ} = e^{-2\mathrm{i}\,\alpha}\phi^{IJ}\,. \tag{3.22}$$

On the other hand, the ω-deformed $\mathcal{N}=6$ theory is obtained by acting with a matrix $\hat{W}$ on $\mathcal{V}$ from the right, where

$$\hat{W} = \begin{pmatrix} e^{\mathrm{i}\,\omega}\mathbb{1}_{16} & 0\\ 0 & e^{-\mathrm{i}\,\omega}\mathbb{1}_{16}\end{pmatrix}. \tag{3.23}$$

Since $\hat{W} \neq \hat{V}$, we see that the ω deformation of the $\mathcal{N}=6$ theory cannot be absorbed by means of any $\mathrm{U}(1)_R$ transformation.

It is, perhaps, worth emphasising that although the above argument shows that ω is a non-trivial parameter in the $\mathcal{N}=6$ theory, it is for the slightly subtle reason that one cannot perform any local field redefinition that would implement a duality rotation on the U(1) gauge field in the Lagrangian itself. One could, of course, make such a duality rotation at the level of the equations of motion. To see this, let us consider the duality rotation

$$\begin{pmatrix} F_{\mu\nu} \\ G_{\mu\nu} \end{pmatrix} \longrightarrow \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} F_{\mu\nu} \\ G_{\mu\nu} \end{pmatrix}, \tag{3.24}$$

where $G_{\mu\nu} = 4\epsilon_{\mu\nu\alpha\beta}\partial\mathcal{L}/\partial F_{\alpha\beta}$, which then implies

$$F^{+}_{1\mu\nu} \longrightarrow e^{-\mathrm{i}\,\beta}\, F^{+}_{1\mu\nu}\,, \qquad F^{+}_{2\mu\nu} \longrightarrow e^{\mathrm{i}\,\beta}\, F^{+}_{2\mu\nu}\,. \tag{3.25}$$

Thus on the full set of $\mathrm{SO}(6)\times\mathrm{U}(1)$ gauge field strength and their duals, this duality transformation is implemented by the matrix

$$\hat{Z} = \begin{pmatrix} Z_{16\times16} & 0 \\ 0 & Z^{*}_{16\times16} \end{pmatrix}, \qquad Z_{16\times16} = \begin{pmatrix} \mathbb{1}_{15} & 0 \\ 0 & e^{-\mathrm{i}\,\beta} \end{pmatrix}. \tag{3.26}$$

It is now evident that if we right-multiply $\mathcal{V}$ by the matrix $\hat{V}\hat{Z}$, with $\alpha=\omega$ and $\beta=-4\omega$, we obtain the same result as the right-multiplication by $\hat{W}$ that generates the ω deformation of the $\mathcal{N}=6$ theory [10]. Although it might therefore appear that the ω deformation is trivial in the $\mathcal{N}=6$ theory this is in fact not the case, since the theory at the quantum level is specified not by its equations of motion but rather, by its Lagrangian, and in the Lagrangian one would have to make a non-local field redefinition of U(1) gauge potential in order to implement the $\hat{V}\hat{Z}$ transformation. In particular, since the Lagrangian defines the nature of the correlation functions in the dual theory, the results can, and indeed do, depend on the value of ω. Nonetheless, As we shall see in section 6, there exists a relationship between the correlation functions of the $\omega=0$ and $\omega=\pi/8$ theories. It is also worth emphasising that since the equations of motion in the ω-deformed $\mathcal{N}=6$ supergravity are independent of the ω parameter, upon the use of a duality rotation of the U(1) gauge field, this implies that the scalar potential V must be independent of ω.

4 Supersymmetric boundary conditions in the ω-deformed $\mathcal{N}=6$ theory

In order to calculate the holographic correlation functions it is crucial to understand the boundary conditions imposed on the fields. In the standard AdS/CFT dictionary, the partition function of the bulk gravity theory is equal to the partition function of the boundary CFT. The boundary values of the bulk fields are identified as the external sources coupling to certain operators on the boundary. In general, the bulk fields satisfy second-order equations and therefore, for a single field, near the boundary of AdS there are two boundaries values associated with different falloff rates. Under certain circumstance, these two boundary values allow the option of different boundary condition choices for the bulk fields.

JHEP02(2015)112

Interestingly, imposing different boundary conditions for the bulk fields leads to different dual boundary CFTs. As an example, in $D = d+1$-dimensional AdS space of unit radius with the metric

$$ds^2 = \frac{dz_0^2 + \sum_{i=1}^d dz_i^2}{z_0^2}\,, \tag{4.1}$$

a scalar field with mass-squared m^2 behaves as $\phi(z) \sim z_0^{\Delta_-}\phi_-(\vec{z}) + z_0^{\Delta_+}\phi_+(\vec{z})$ when approaching the AdS boundary, where $\Delta_\pm = \frac{1}{2}(d \pm \sqrt{d^2+4m^2})$. It was established in [11–13] that for $m^2 > -\frac{d^2}{4}+1$, there is a unique admissible boundary condition, for which $\phi_-(\vec{z})$ is treated as the external source on the boundary CFT. However, for $-\frac{d^2}{4} < m^2 < -\frac{d^2}{4}+1$ there are additional possible conditions. One can impose either a Neumann boundary condition by identifying ϕ_+ as an external source on the boundary, or else a mixed condition by imposing a functional relation between ϕ_- and ϕ_+. The Neumann boundary condition leads to an alternative quantization on the boundary CFT [14], while the mixed boundary condition is interpreted as deforming the CFT by multi-trace operators [15, 16].

For bulk Abelian gauge vectors, it was shown in [17] that for $d = 3$, 4 and 5, both the slow-falloff and the fast-falloff parts of the vector are normalizable. Furthermore, in [18] it was suggested that if the Neumann boundary condition is adopted for the vector field, the dual of the vector field represents a dynamical gauge field in the CFT. Similarly, the possibility of imposing different boundary conditions for spin-$\frac{3}{2}$ and for the graviton has been explored in [18–20]. The Neumann boundary conditions for the gravitino and the graviton imply the existence of dynamical gravitino or graviton fields in the boundary theory.

In this work, we shall focus on the case where the ω-deformed $\mathcal{N}=6$ gauged supergravity can still have a dual CFT description. We shall, therefore, still impose standard Dirichlet boundary conditions on the spin-$\frac{3}{2}$ and spin-2 gauge fields. With the boundary conditions for the spin-$\frac{3}{2}$ and spin-2 fields thus determined, the supersymmetry-preserving boundary conditions for the lower-spin fields can then be derived from the supersymmetry transformations of the coefficient functions associated with the large-distance expansions of the lower-spin fields [20–22].

Turning now to the specific case of the ω-deformed gauged $\mathcal{N}=6$ supergravity, we therefore begin, as discussed above, by imposing Dirichlet boundary condition on the spin-2 and spin-$\frac{3}{2}$ fields. From the supersymmetry variation of the leading coefficients of the spin-2 and spin-$\frac{3}{2}$ fields[1]

$$\begin{aligned}\delta e^{\hat{r}}_{(0)i} &= \bar{\epsilon}^I_+ \gamma^{\hat{r}}_{(0)} \psi^I_{(0)i+}\,,\\ \delta\psi^I_{(0)i+} &= \frac{1}{2}\mathcal{K}^{ab}_{(0)i}\gamma_{ab}\epsilon^I_+ + \sqrt{2}A^{IJ}_{(0)i}\epsilon^J_+\,,\end{aligned} \tag{4.2}$$

we then deduce that the vanishing of $\psi^I_{(0)i+}$ implies $A^{IJ}_{(0)i} = 0$. The supersymmetry variation of $A^{IJ}_{(0)i}$, given by

$$\begin{aligned}\delta A^{IJ}_{(0)i} &= -\Big(\cos 2\omega \bar{\epsilon}^K_+ \gamma_{(0)i}\chi^{IJK}_+ + \mathrm{i}\sin 2\omega \bar{\epsilon}^K_+ \gamma_{(0)i}\gamma_5\chi^{IJK}_-\Big) + \cdots\,,\\ \delta A^{IJ}_{(1)i} &= -\mathcal{S}^{IJKL}_{(1)}\bar{\epsilon}^M_+ \gamma_{(0)i}\chi^{KLM}_+ - \mathrm{i}\,\mathcal{P}^{IJKL}_{(1)}\bar{\epsilon}^M_+\gamma_{(0)i}\gamma_5\chi^{KLM}_- - 2\sqrt{2}\bar{\epsilon}^{[I}_+\psi^{J]}_{i(3)-}\end{aligned}$$

[1]For simplicity, we work with AdS_4 of unit radius, which corresponds to setting $g = 1/\sqrt{2}$.

$$+D_i(A_{(0)})\Big(\cos 2\omega\bar{\epsilon}_{K+}\chi_-^{IJK} + \mathrm{i}\sin 2\omega\bar{\epsilon}_{K+}\gamma_5\chi_+^{IJK}\Big) + \cdots\,, \tag{4.3}$$

will then require[2]

$$\cos 2\omega\,\chi_+^{IJK} + \mathrm{i}\sin 2\omega\,\gamma_5\chi_-^{IJK} = 0\,. \tag{4.4}$$

The variation of the above equation should also vanish. Using

$$\begin{aligned}\delta\chi_+^{IJK} &= -\mathcal{S}_{(2)}^{IJKL}\epsilon_+^L + 2\mathrm{i}\,\mathcal{P}_{(1)}^{IJKL}\gamma_5\epsilon_-^L - \mathrm{i}\,\not{D}\mathcal{P}_{(1)}^{IJKL}\gamma_5\epsilon_+^L\,,\\ \delta\chi_-^{IJK} &= 2\mathcal{S}_{(1)}^{IJKL}\epsilon_-^L - \mathrm{i}\mathcal{P}_{(2)}^{IJKL}\gamma_5\epsilon_+^L + \not{D}\mathcal{S}_{(1)}^{IJKL}\epsilon_+^L\,,\end{aligned} \tag{4.5}$$

we deduce that

$$\begin{aligned}&\cos 2\omega\mathcal{P}_{(1)}^{IJKL} + \sin 2\omega\mathcal{S}_{(1)}^{IJKL} = 0 \Rightarrow -\cos 2\omega\mathcal{P}_{(1)}^{IJ} + \sin 2\omega\mathcal{S}_{(1)}^{IJ} = 0\,,\\ &\cos 2\omega\mathcal{S}_{(2)}^{IJKL} - \sin 2\omega\mathcal{P}_{(2)}^{IJKL} = 0 \Rightarrow \cos 2\omega\mathcal{S}_{(2)}^{IJ} + \sin 2\omega\mathcal{P}_{(2)}^{IJ} = 0\,,\end{aligned} \tag{4.6}$$

which will further imply

$$\cos 2\omega\chi_+^I - \mathrm{i}\sin 2\omega\gamma_5\chi_-^I = 0\,, \tag{4.7}$$

according to the variation of $\mathcal{S}^{IJ}$ and $\mathcal{P}^{IJ}$

$$\begin{aligned}\delta\mathcal{S}_{(1)}^{IJ} &= 2\bar{\epsilon}_+^{[I}\chi_-^{J]} + \frac{1}{3!}\varepsilon^{IJKLMN}\bar{\epsilon}_+^K\chi_-^{LMN}\,,\\ \delta\mathcal{P}_{(1)}^{IJ} &= -2\mathrm{i}\,\bar{\epsilon}_+^{[I}\gamma_5\chi_+^{J]} + \frac{\mathrm{i}}{3!}\varepsilon^{IJKLMN}\bar{\epsilon}_+^K\gamma_5\chi_+^{LMN}\,,\\ \delta\mathcal{S}_{(2)}^{IJ} &= 2\Big(\bar{\epsilon}_-^{[I}\chi_+^{J]} + \bar{\epsilon}_+^{[I}\not{D}\chi_+^{J]}\Big) + \frac{1}{3!}\varepsilon^{IJKLMN}\Big(\bar{\epsilon}_-^K\chi_+^{LMN} + \bar{\epsilon}_+^K\not{D}\chi_+^{LMN}\Big)\,,\\ \delta\mathcal{P}_{(2)}^{IJ} &= -2\mathrm{i}\Big(\bar{\epsilon}_-^{[I}\gamma_5\chi_-^{J]} - \bar{\epsilon}_+^{[I}\gamma_5\not{D}\chi_-^{J]}\Big) + \frac{\mathrm{i}}{3!}\varepsilon^{IJKLMN}\Big(\bar{\epsilon}_-^K\gamma_5\chi_-^{LMN} - \bar{\epsilon}_+^K\gamma_5\not{D}\chi_-^{LMN}\Big)\,.\end{aligned} \tag{4.8}$$

It can be checked that (4.6) and (4.7) are consistent with the variation of χ^I

$$\begin{aligned}\delta\chi_+^I &= -\mathcal{S}_{(2)}^{IJ}\epsilon_+^J + 2\mathrm{i}\,\mathcal{P}_{(1)}^{IJ}\gamma_5\epsilon_-^J - \mathrm{i}\,\not{D}\mathcal{P}_{(1)}^{IJ}\gamma_5\epsilon_+^J,\\ \delta\chi_-^I &= 2\mathcal{S}_{(1)}^{IJ}\epsilon_-^J - \mathrm{i}\mathcal{P}_{(2)}^{IJ}\gamma_5\epsilon_+^J + \not{D}\mathcal{S}_{(1)}^{IJ}\epsilon_+^J\,.\end{aligned} \tag{4.9}$$

Finally, given the boundary conditions for the spin-$\frac{3}{2}$, spin-$\frac{1}{2}$ (4.4), (4.7) and spin-0 fields (4.6), we can derive the admissible boundary condition for the U(1) gauge field that preserve $\mathcal{N}=6$ supersymmetry. Using

$$\begin{aligned}\delta A_{(0)i} &= -\Big(\cos 2\omega\bar{\epsilon}_+^I\gamma_{(0)i}\chi_+^I + \mathrm{i}\sin 2\omega\bar{\epsilon}_+^I\gamma_{(0)i}\gamma_5\chi_-^I\Big) + \cdots\,,\\ \delta A_{(1)i} &= -\mathcal{S}_{(1)}^{IJ}\bar{\epsilon}_+^K\gamma_{(0)i}\widetilde{\chi}_+^{IJK} - \mathrm{i}\,\mathcal{P}_{(1)}^{IJ}\bar{\epsilon}_+^K\gamma_{(0)i}\gamma_5\chi_-^{IJK}\\ &\quad+\partial_i\Big(\cos 2\omega\bar{\epsilon}_{I+}\chi_-^I + \mathrm{i}\sin 2\omega\bar{\epsilon}_{I+}\gamma_5\chi_+^I\Big) + \cdots\,,\end{aligned} \tag{4.10}$$

one can see that when $\omega\neq 0$, the only boundary condition preserving the $\mathcal{N}=6$ supersymmetry is given by

$$A_{(1)\mu} = 0, \qquad \omega = \frac{\pi}{8}\,. \tag{4.11}$$

[2]The ellipses refer to term depending on $\psi^r_{(0)i+}$ which vanish for the Dirichlet boundary conditions that we impose on the gravitini.

In other words, the only case within the class of ω-deformed $\mathcal{N}=6$ supergravities where there can exist consistent boundary conditions that preserve the $\mathcal{N}=6$ supersymmetry is when $\omega=\pi/8$. The U(1) gauge field must then satisfy the Neumann, rather than Dirichlet, boundary condition.

If we redefine the complex scalar

$$\widetilde{\varphi}^{IJ}\equiv e^{-2\mathrm{i}\,\omega}\varphi^{IJ}\,, \tag{4.12}$$

the $\mathcal{N}=6$ boundary condition can be summarized as

$$\begin{aligned}&e_i^{\hat{r}}=\delta_i^{\hat{r}}\,,\ \psi^I_{(0)i}=0\,,\ A^{IJ}_{(0)i}=0,\ A_{(1)i}=0,\ \widetilde{\mathcal{S}}^{IJ}_{(1)}=0\,,\ \widetilde{\mathcal{P}}^{IJ}_{(2)}=0\,,\\&\cos 2\omega\chi_+^{IJK}+\mathrm{i}\sin 2\omega\gamma_5\chi_-^{IJK}=0\,,\quad \cos 2\omega\chi_+^{I}-\mathrm{i}\sin 2\omega\gamma_5\chi_-^{I}=0\,,\end{aligned} \tag{4.13}$$

with the condition also that $\omega=\pi/8$.

5 ω-dependent 3-point tree graphs

One of our primary goals in this paper is to identify and compute the simplest correlation functions that are sensitive to the ω-deformations of the bulk supergravity theory, in order to gain insights into the effects of the deformations on the boundary conformal field theory. Our starting point is to study the expansion of the four-dimensional fields around the trivial AdS_4 vacuum of the $\mathcal{N}=8$ ω-deformed gauged supergravity. For the reasons discussed already, our focus will be on the consistent truncation of the $\mathcal{N}=8$ theory to $\mathcal{N}=6$, for which, as we showed in section 4, we can impose supersymmetric boundary conditions in which the graviton and gravitini obey standard Dirichlet asymptotics. The scalar potential of the $\mathcal{N}=6$ truncation is independent of the deformation parameter ω, and so within the bosonic sector this leaves the coupling of the scalars in the gauge-field kinetic terms as the remaining place where ω-dependence can enter. In appendix A, we present the expansion of the the gauge-field kinetic terms of the ω-deformed $\mathcal{N}=8$ theory up to quartic order in the bosonic fields. From this expansion, it is evident that there is in fact ω dependence in the trilinear couplings of two gauge fields with a scalar field, and thus for our present purposes it suffices to focus on these terms of cubic order in the bosonic fields.

The ω dependence in the $\varepsilon_{IJKLMN}\varphi^{IJ}F^{KL}F^{MN}$ terms can be absorbed into a redefinition of φ^{IJ}, as in (4.12). A further advantage of using the redefined $\widetilde{\varphi}^{IJ}$ is that the real and imaginary parts of $\widetilde{\varphi}^{IJ}$ then satisfy definite boundary conditions (4.13), whereas the boundary conditions imposed on the original scalars φ^{IJ} involve ω-dependent linear combinations of the real and imaginary parts of φ^{IJ}.

After taking the redefinition into account, we are left with two ω-dependent vertices at the trilinear order. After truncating to the fields of $\mathcal{N}=6$ supergravity, as discussed in section 3, these are given by

$$\begin{aligned}&1)\qquad -\sqrt{2}\int\frac{d^4z}{z_0^4}\mathrm{Re}(e^{4\mathrm{i}\omega}\,\widetilde{\varphi}^{IJ})\partial_{[\mu}A_{\nu]}{}^{IJ}\,\partial_\mu A_\nu\,,\\&2)\qquad -\frac{\mathrm{i}}{\sqrt{2}}\int\frac{d^4z}{z_0^4}\mathrm{Im}(e^{4i\omega}\,\widetilde{\varphi}^{IJ})\,\epsilon_{\mu\nu\rho\sigma}\partial_\mu A_\nu{}^{IJ}\partial_\rho A_\sigma\,.\end{aligned} \tag{5.1}$$

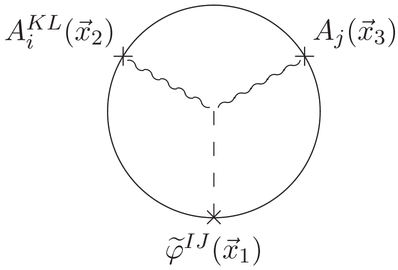

Figure 1. Witten diagram corresponding to the ω-dependent bulk 3-point interactions.

In the above expressions, we do not distinguish the lower and upper indices since in this section we work with Euclidean signature. As we discussed in the previous section, for non-zero values of the ω parameter $\mathcal{N}=6$ supersymmetry of the boundary conditions requires $\omega=\pi/8$, and specific asymptotic falloff behaviors for the scalars near the boundary of AdS. Explicitly, $\widetilde{\mathcal{S}}^{IJ}(z)\sim z_0^2\widetilde{\mathcal{S}}^{IJ}(\vec{z})$ and $\widetilde{\mathcal{P}}^{IJ}(z)\sim z_0\widetilde{\mathcal{P}}^{IJ}(\vec{z})$, which means $\Delta=2$ in the bulk to boundary propagator of $\widetilde{\mathcal{S}}^{IJ}$ and $\Delta=1$ in the bulk to boundary propagator of $\widetilde{\mathcal{P}}^{IJ}$. We shall, however, keep our discussion more general, and leave the conformal dimension Δ for the scalars $\widetilde{\mathcal{S}}^{IJ}$ and $\widetilde{\mathcal{P}}^{IJ}$ arbitrary for now.

According to the AdS/CFT dictionary, the cubic interactions in the supergravity Lagrangian are associated with certain 3-point correlation functions in the boundary CFT. The mapping from the bulk interaction to the correlators on the boundary is represented by the Witten diagram:

The 3-point amplitude corresponding to the first bulk cubic interaction in (5.1) can be expressed as

$$\begin{aligned}T_{ij}^{(1)IJ,KL}(\vec{x}_1,\vec{x}_2,\vec{x}_3) = &-\frac{1}{\sqrt{2}}(\delta^{IK}\delta^{JL}-\delta^{IL}\delta^{JK})\\ &\times\int\frac{dz_0d^3\vec{z}}{z_0^4}K_\Delta(z,\vec{x}_1)g^{\mu\rho}g^{\nu\sigma}\partial_{[\mu}G_{\nu]i}(z,\vec{x}_2)\partial_\rho\widetilde{G}_{\sigma j}(z,\vec{x}_3)\,,\end{aligned}\tag{5.2}$$

in which the $K_\Delta(z,\vec{x})$ is the bulk to boundary propagator associated with the scalar field of conformal dimension Δ,

$$K_\Delta(z,\vec{x}) = c_\Delta\left(\frac{z_0}{z_0^2+(\vec{z}-\vec{x})^2}\right)^\Delta,\qquad c_\Delta=\frac{\Gamma(\Delta)}{\pi^{\frac{3}{2}}\Gamma(\Delta-\frac{3}{2})}\,.\tag{5.3}$$

$G_{\mu i}(z,\vec{x})$ is the bulk to boundary propagator associated with A_i^{IJ}. Since A_i^{IJ} satisfies Dirichlet boundary conditions, its bulk to boundary propagator takes the standard form [24]

$$G_{\mu i}(z,\vec{x}) = c_3\frac{z_0}{[z_0^2+(\vec{z}-\vec{x})^2]^2}J_{\mu i}(z-\vec{x}),\quad c_3=\frac{2}{\pi^2}\,,\tag{5.4}$$

where

$$J_{\mu\nu}(x-y)=\delta_{\mu\nu}-\frac{2(x-y)_\mu\,(x-y)_\nu}{|x-y|^2}\,.\tag{5.5}$$

JHEP02(2015)112

It follows that $G_{\mu i}(z,\vec{x})$ satisfies

$$\partial_{[\mu}G_{\nu]i}(z,\vec{x}) = \frac{2}{\pi^2[z_0^2+(\vec{z}-\vec{x})^2]^2}\,J_{0[\mu}(z-\vec{x})J_{\nu]i}(z-\vec{x})\,. \tag{5.6}$$

It can also be checked that

$$\frac{\partial}{\partial x_i}\partial_{[\mu}G_{\nu]i}(z,\vec{x}) = 0\,, \tag{5.7}$$

which implies that A_i^{IJ} is dual to a conserved current of dimension 2. $\widetilde{G}_{\mu i}(z,\vec{x})$ is the bulk to boundary propagator associated with A_i.

Since the bulk U(1) gauge field A_i satisfies instead a Neumann boundary condition, its bulk to boundary propagator takes a different form [23]. It is shown in [23] that up to a pure gauge term, the propagator for a U(1) gauge field in AdS$_{d+1}$ can be expressed as

$$G_{\mu\nu}(z,w) = -F(u)\,\frac{\partial^2 u}{\partial z^\mu \partial w^\nu}\,, \qquad u \equiv \frac{(z-w)^2}{2z_0w_0}\,, \tag{5.8}$$

where $F(u)$ satisfies

$$u(u+2)F'' + (d+1)(1+u)F' + (d-1)F = 0\,. \tag{5.9}$$

Up to a proportionality constant, the two independent solutions of this equation are given by

$$F_1(u) = \frac{1}{u(u+2)}\,, \qquad F_2(u) = \frac{1}{u}\,. \tag{5.10}$$

The propagator associated with the usual Dirichlet boundary condition can be obtained by choosing $F(u) \propto F_1$. If instead $F(u)$ is chosen to be proportional to $F_2(u)$, we obtain the U(1) gauge boson propagator associated with the Neumann boundary condition, which we shall denote by $\widetilde{G}_{\mu\nu}$. It is given by

$$\widetilde{G}_{\mu i}(z,\vec{x}) = \tilde{c}_3\frac{1}{[z_0^2+(\vec{z}-\vec{x})^2]}(\delta_{\mu i} + \frac{(\vec{x}-\vec{z})_i}{z_0}\delta_{\mu 0}), \quad \tilde{c}_3 = -\frac{1}{2\pi^2}\,, \tag{5.11}$$

with its curl given by

$$\partial_{[\mu}\widetilde{G}_{\nu]i}(z,\vec{x}) = -\frac{1}{2\pi^2 z_0[z_0^2+(\vec{z}-\vec{x})^2]}J_{0[\mu}(z-\vec{x})J_{\nu]i}(z-\vec{x})\,. \tag{5.12}$$

The constant $\tilde{c}_3$ is fixed by requiring that $\widetilde{G}_{\mu i}(z,\vec{x}) \to z_0\delta_{ij}\delta^3(\vec{z}-\vec{x})$ when $z_0 \to 0$.

It should be noted that unlike the Dirichlet propagator $G_{\mu i}(z,\vec{x})$, the Neumann propagator has

$$\frac{\partial}{\partial x_i}\partial_{[\mu}\widetilde{G}_{\nu]i}(z,\vec{x}) \neq 0\,. \tag{5.13}$$

This means that the holographic dual of A_i cannot be a conserved current. On the other hand, the 2-point amplitude associated with two A_i fields reveals that the dual of A_i should have dimension 1, which is below the unitary bound for a spin-1 operator in CFT$_3$. However, 1 is the correct dimension for a Chern-Simons gauge field. Following [18], these facts lead to the conclusion that the holographic dual of A_i is a dynamical U(1) Chern-Simons gauge field.

Because of the translation invariance in the 3-dimensional boundary directions, the 3-point amplitude derived from the first cubic vertex depends only on the difference of boundary coordinates. Thus

$$T_{ij}^{(1)IJ,KL}(\vec{x}_1,\vec{x}_2,\vec{x}_3) = T_{ij}^{(1)IJ,KL}(\vec{x}_{13},\vec{x}_{23},0)\,, \tag{5.14}$$

where $\vec{x}_{13}=\vec{x}_1-\vec{x}_3$ and $\vec{x}_{23}=\vec{x}_2-\vec{x}_3$. For simplicity, we shall first compute the amplitude

$$T_{ij}^{(1)IJ,KL}(\vec{w},\vec{x},0)\,, \tag{5.15}$$

and later replace $\vec{w}$ by $\vec{x}_{13}$ and $\vec{x}$ by $\vec{x}_{23}$. To compute (5.15), we follow the strategy of [24] by expressing (5.15) as

$$T_{ij}^{(1)IJ,KL}(\vec{w},\vec{x},\vec{y})|_{|y|\to 0}\,. \tag{5.16}$$

Then we use the inversion trick,

$$z_\mu = \frac{z'_\mu}{z'^2}, \qquad \vec{w} = \frac{\vec{w}'}{|w'|^2}, \qquad \vec{x} = \frac{\vec{x}'}{|x'|^2}, \qquad \vec{y} = \frac{\vec{y}'}{|y'|^2}\,. \tag{5.17}$$

Under the inversion of coordinates, the propagators transform covariantly as

$$\begin{aligned}
K_\Delta(z,\vec{w}) &= K_\Delta(z',\vec{w}')|w'|^{2\Delta}\,,\\
\partial_{[\mu}G_{\nu]i}(z,\vec{x}) &= (z')^2J_{\mu\rho}(z')\cdot(z')^2J_{\nu\sigma}(z')\cdot(\vec{x}')^4J_{ki}(\vec{x})\cdot\partial'_{[\rho}G_{\sigma]k}(z',\vec{x}')\,,\\
\partial_{[\mu}\widetilde{G}_{\nu]i}(z,\vec{x}) &= (z')^2J_{\mu\rho}(z')\cdot(z')^2J_{\nu\sigma}(z')\cdot(\vec{x}')^2J_{ki}(\vec{x})\cdot\partial'_{[\rho}\widetilde{G}_{\sigma]k}(z',\vec{x}')\,.
\end{aligned} \tag{5.18}$$

After some algebra, the scalar integral in (5.2) can be simplified to give

$$\begin{aligned}
&c_\Delta\tilde{c}_3\int dz'_0d^3\vec{z}'\frac{z_0'^{\Delta}}{[z_0'^2+(\vec{z}'-\vec{w}')^2]^\Delta}(\vec{x}')^4J_{ki}(\vec{x}')\partial'_{[0}G_{j]k}(z',\vec{x}')\frac{1}{z'_0}(\vec{w}')^{2\Delta}\\
&= -\frac{\Gamma(\Delta)}{\pi^{\frac{11}{2}}\Gamma(\Delta-\frac{3}{2})}(\vec{x}')^4(\vec{w}')^{2\Delta}J_{ki}(\vec{x}')\\
&\quad\times\int\frac{dz'_0d^3\vec{z}'}{z'_0}\left(\frac{z'_0}{(z'-\vec{w}')^2}\right)^\Delta\frac{\partial}{\partial z'_{[0}}\left(\frac{z'_0}{(z'-\vec{x}')^2}\right)\frac{\partial}{\partial z'_{j]}}\left(\frac{(z'-\vec{x}')_k}{(z'-\vec{x}')^2}\right).
\end{aligned} \tag{5.19}$$

The above expression can be computed by using two integral formulae. The first is given in [24], namely

$$\begin{aligned}
&\int_0^\infty dz_0\int d^3\vec{z}\frac{z_0^a}{[z_0^2+(\vec{z}-\vec{x})^2]^b[z_0^2+(\vec{z}-\vec{y})^2]^c} \equiv I[a,b,c,3]|\vec{x}-\vec{y}|^{4+a-2b-2c},\\
&I[a,b,c,3] = \frac{\pi^{\frac{3}{2}}}{2}\frac{\Gamma[\frac{a+1}{2}]\Gamma[b+c-\frac{a}{2}-2]\Gamma[2+\frac{a}{2}-b]\Gamma[2+\frac{a}{2}-c]}{\Gamma[b]\Gamma[c]\Gamma[4+a-b-c]}\,.
\end{aligned} \tag{5.20}$$

The second takes the form

$$\int_0^\infty dz_0\int d^3\vec{z}\frac{z_0^a(\vec{z}-\vec{y})_i}{[z_0^2+(\vec{z}-\vec{x})^2]^b[z_0^2+(\vec{z}-\vec{y})^2]^c} = \widetilde{I}[a,b,c,3]\frac{(\vec{x}-\vec{y})_i}{|\vec{x}-\vec{y}|^{2b+2c-a-4}}\,,$$

$$\widetilde{I}[a,b,c,3]=\frac{\pi^{\frac{3}{2}}}{2}\frac{\Gamma[\frac{a+1}{2}]\Gamma[3+\frac{a}{2}-c]\Gamma[2+\frac{a}{2}-b]\Gamma[b+c-\frac{a}{2}-2]}{\Gamma[b]\Gamma[c]\Gamma[5+a-b-c]}\,. \tag{5.21}$$

In fact for $c>2$ the second integral can be expressed as the derivative of the first with respect to y_i. For generic values of c, integral (5.21) can be directly computed by using Feynman integral techniques with two denominators. After performing the z integral, we obtain

$$\begin{aligned}
T^{(1)IJ,KL}_{ij}(\vec{w},\vec{x},0) &= \frac{\Gamma[\frac{1+\Delta}{2}]^2\Gamma[\frac{\Delta}{2}]\Gamma[\frac{3-\Delta}{2}]}{16\sqrt{2}\pi^4\Gamma[\Delta-\frac{3}{2}]}(\delta^{IK}\delta^{JL}-\delta^{IL}\delta^{JK})\\
&\quad\times\Big((\Delta-3)J_{ij}(\vec{x}')+(\Delta+1)J_{ik}(\vec{x}')J_{kj}(\vec{w}'-\vec{x}')\Big)\frac{|\vec{x}'|^4|\vec{w}'|^{2\Delta}}{|\vec{w}'-\vec{x}'|^{\Delta+1}}\\
&= \frac{\Gamma[\frac{1+\Delta}{2}]^2\Gamma[\frac{\Delta}{2}]\Gamma[\frac{3-\Delta}{2}]}{16\sqrt{2}\pi^4\Gamma[\Delta-\frac{3}{2}]}(\delta^{IK}\delta^{JL}-\delta^{IL}\delta^{JK})\\
&\quad\times\frac{(\Delta-3)J_{ij}(\vec{x})+(\Delta+1)J_{ik}(\vec{x}-\vec{w})J_{kj}(\vec{w})}{|\vec{x}|^{3-\Delta}|\vec{w}|^{\Delta-1}|\vec{w}-\vec{x}|^{\Delta+1}}\,.
\end{aligned} \tag{5.22}$$

It can be checked that

$$\frac{\partial}{\partial x_j}T^{(1)IJ,KL}_{ij}(\vec{w},\vec{x},0)=0\,. \tag{5.23}$$

In terms of the original $\vec{x}_i$ coordinates,

$$\begin{aligned}
T^{(1)IJ,KL}_{ij}(\vec{x}_1,\vec{x}_2,\vec{x}_3) &= \frac{\Gamma[\frac{1+\Delta}{2}]^2\Gamma[\frac{\Delta}{2}]\Gamma[\frac{3-\Delta}{2}]}{16\sqrt{2}\pi^4\Gamma[\Delta-\frac{3}{2}]}(\delta^{IK}\delta^{JL}-\delta^{IL}\delta^{JK})\\
&\quad\times\frac{(\Delta-3)J_{ij}(\vec{x}_2-\vec{x}_3)+(\Delta+1)J_{ik}(\vec{x}_2-\vec{x}_1)J_{kj}(\vec{x}_1-\vec{x}_3)}{|\vec{x}_2-\vec{x}_3|^{3-\Delta}|\vec{x}_1-\vec{x}_3|^{\Delta-1}|\vec{x}_1-\vec{x}_2|^{\Delta+1}}\,.
\end{aligned} \tag{5.24}$$

The second ω-dependent cubic vertex in (5.1) leads to another 3-point boundary amplitude,

$$\begin{aligned}
T^{(2)IJ,KL}_{ij}(\vec{x}_1,\vec{x}_2,\vec{x}_3) &= -\frac{\mathrm{i}}{2\sqrt{2}}(\delta^{IK}\delta^{JL}-\delta^{IL}\delta^{JK})\\
&\quad\times\int\frac{dz_0d^3\vec{z}}{z_0^4}K_\Delta(z,\vec{x}_1)\epsilon^{\mu\nu\rho\sigma}\partial_\mu G_{\nu i}(z,\vec{x}_2)\partial_\rho\widetilde{G}_{\sigma j}(z,\vec{x}_3)\,.
\end{aligned} \tag{5.25}$$

Following the same strategy, we shall compute

$$T^{(2)IJ,KL}_{ij}(\vec{w},\vec{x},0)\,, \tag{5.26}$$

and replace w by $\vec{x}_{13}$ and x by $\vec{x}_{23}$ at the final stage of the calculation. Utilizing again the inversion trick and the properties of the various propagators under the inversion of coordinates, the scalar integral in the amplitude (5.25) can be expressed as

$$\begin{aligned}
&c_\Delta\tilde{c}_3\int\frac{dz_0'd^3\vec{z}'}{z_0'^4}\frac{z_0'^\Delta}{[z_0'^2+(\vec{z}'-\vec{w}')^2]^\Delta}\epsilon^{mnj0}(\vec{x}')^4J_{ki}(\vec{x}')\partial_m'G_{nk}(z',\vec{x}')\frac{1}{z_0'}(\vec{w}')^{2\Delta}\\
&=\frac{2\Gamma(\Delta)\varepsilon^{mkj0}}{\pi^{\frac{11}{2}}\Gamma(\Delta-\frac{3}{2})}\int dz_0'd^3\vec{z}'\frac{z_0'^\Delta}{[z_0'^2+(\vec{z}'-\vec{w}')^2]^\Delta}\frac{(\vec{x}')^4J_{ki}(\vec{x}')(\vec{z}'-\vec{x}')_m(\vec{w}')^{2\Delta}}{[z_0'^2+(\vec{z}'-\vec{x}')^2]^3}\,.
\end{aligned} \tag{5.27}$$

In the derivation of the above results, the following equalities have been used

$$\begin{aligned} \epsilon^{\mu\nu\rho\sigma} J_{\mu\lambda}(z') J_{\nu\tau}(z') J_{\rho\gamma}(z') J_{\sigma\eta}(z') &= -\epsilon^{\lambda\tau\gamma\eta} \,, \\ J_{lj}(\vec{y}') J_{\eta l}(\vec{z}' - \vec{y}') J_{0\gamma}(\vec{z}' - \vec{y}')|_{|y'|\to\infty} &= \delta_{j\eta}\delta_{0\gamma} \,. \end{aligned} \tag{5.28}$$

Using the previous integral formulae, we obtain

$$\begin{aligned} T^{(2)IJ,KL}_{ij}(\vec{w},\vec{x},0) &= -\mathrm{i}\,\frac{\Gamma[\frac{1+\Delta}{2}]\Gamma[\frac{\Delta}{2}]^2\Gamma[2-\frac{\Delta}{2}]}{8\sqrt{2}\pi^4\Gamma[\Delta-\frac{3}{2}]}(\delta^{IK}\delta^{JL}-\delta^{IL}\delta^{JK}) \\ &\quad \times (\vec{x}')^4(\vec{w}')^{2\Delta}\varepsilon_{mkj}J_{ki}(\vec{x}')\partial_{x'_m}|\vec{w}'-\vec{x}'|^{-\Delta} \\ &= -\mathrm{i}\,\frac{\Delta\Gamma[\frac{1+\Delta}{2}]\Gamma[\frac{\Delta}{2}]^2\Gamma[2-\frac{\Delta}{2}]}{8\sqrt{2}\pi^4\Gamma[\Delta-\frac{3}{2}]}(\delta^{IK}\delta^{JL}-\delta^{IL}\delta^{JK}) \\ &\quad \times \frac{|\vec{w}|^{2-\Delta}}{|\vec{w}-\vec{x}|^{\Delta}|\vec{x}|^{4-\Delta}}\varepsilon_{mkj}J_{ki}(\vec{x})J_{m\ell}(\vec{w})\left[\frac{(w-x)_\ell}{|\vec{w}-\vec{x}|^2}-\frac{w_\ell}{|\vec{w}|^2}\right]. \end{aligned} \tag{5.29}$$

In terms of the original x_i coordinates, the second 3-point amplitude is given by

$$\begin{aligned} T^{(2)IJ,KL}_{ij}(\vec{x}_1,\vec{x}_2,\vec{x}_3) &= \mathrm{i}\,\frac{\Gamma[\frac{1+\Delta}{2}]\Gamma[\frac{\Delta}{2}]\Gamma[\frac{\Delta}{2}+1]\Gamma[2-\frac{\Delta}{2}]}{4\sqrt{2}\pi^4\Gamma[\Delta-\frac{3}{2}]}(\delta^{IK}\delta^{JL}-\delta^{IL}\delta^{JK}) \\ &\times \frac{|\vec{x}_1-\vec{x}_3|^{2-\Delta}\varepsilon_{mkj}}{|\vec{x}_1-\vec{x}_2|^{\Delta+2}|\vec{x}_2-\vec{x}_3|^{2-\Delta}}J_{ki}(\vec{x}_2-\vec{x}_3)\left[\frac{(x_3-x_1)_m}{|\vec{x}_3-\vec{x}_1|^2}-\frac{(x_3-x_2)_m}{|\vec{x}_3-\vec{x}_2|^2}\right]. \end{aligned} \tag{5.30}$$

To arrive at the result above, we have used a useful formula given below

$$J_{m\ell}(\vec{x}_3-\vec{x}_1)\left[\frac{(x_1-x_2)_\ell}{|\vec{x}_1-\vec{x}_2|^2}-\frac{(x_1-x_3)_\ell}{|\vec{x}_1-\vec{x}_3|^2}\right] = -\frac{|\vec{x}_3-\vec{x}_2|^2}{|\vec{x}_1-\vec{x}_2|^2}\left[\frac{(x_3-x_1)_m}{|\vec{x}_3-\vec{x}_1|^2}-\frac{(x_3-x_2)_m}{|\vec{x}_3-\vec{x}_2|^2}\right]. \tag{5.31}$$

6 Interpretation in the dual theory

In this section we shall discuss how our results for amplitudes in the ω-deformed bulk theory at $\omega = \pi/8$ are related to a certain operation on the $\mathrm{U}(1)\times\mathrm{U}(1)$ sector of the $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ ABJM theory. In the bulk, we shall denote the U(1) gauge field in the $\omega = 0$ theory by A'_μ, to distinguish it from A_μ in the deformed theory.

The holographic 2-point function associated the U(1) bulk gauge field for the $\omega = 0$ case is given by [24],

$$\left.\frac{\delta^2 S[A']}{\delta A'_{(0)i}(\vec{x}_1)\,\delta A'_{(0)j}(\vec{x}_2)}\right|_{\omega=0} = \langle J_i(\vec{x}_1)J_j(\vec{x}_2)\rangle\Big|_{\omega=0} = \frac{1}{\pi^2}\frac{J_{ij}}{|\vec{x}_1-\vec{x}_2|^4}\,. \tag{6.1}$$

In the $\omega = \pi/8$ theory, recalling that the bulk U(1) gauge field obeys a Neumann boundary condition, we have

$$\left.\frac{\delta^2 S[A]}{\delta A_{(1)i}(\vec{x}_1)\,\delta A_{(1)j}(\vec{x}_2)}\right|_{\omega=\pi/8} = \langle A_{(0)i}(\vec{x}_1)A_{(0)j}(\vec{x}_2)\rangle\Big|_{\omega=\pi/8} = \frac{1}{4\pi^2}\frac{\delta_{ij}}{|\vec{x}_1-\vec{x}_2|^2}\,. \tag{6.2}$$

It is easy to check that

$$\langle J^{\rm top}_i(\vec{x}_1)J^{\rm top}_j(\vec{x}_2)\rangle\Big|_{\omega=\pi/8} = \langle J_i(\vec{x}_1)J_j(\vec{x}_2)\rangle\Big|_{\omega=0}\,, \tag{6.3}$$

where we have defined the topological current to be

$$J_i^{\rm top} = {\rm i}\,\varepsilon_{ijk}\partial_j A_{(0)k}\,. \tag{6.4}$$

This relation can be understood as the electric-magnetic rotation in the bulk theory as follows. From (3.24), it can be seen that the U(1) gauge fields in $\omega = 0$ and $\omega = \pi/8$ theories are related on-shell by

$$F'_{\mu\nu} = -G_{\mu\nu}\,, \tag{6.5}$$

where, in Minkowski signature, we have

$$G_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F^{\alpha\beta} + \frac{1}{\sqrt{2}}{\rm Re}(e^{2{\rm i}\,\omega}\phi^{IJ})\epsilon_{\mu\nu\alpha\beta}F^{IJ\alpha\beta} + \frac{1}{\sqrt{2}}{\rm Im}(e^{2{\rm i}\,\omega}\phi^{IJ})F^{IJ}_{\mu\nu} + \cdots\,, \tag{6.6}$$

where the ellipses denote terms of higher order in the scalar fields (see eqn (2.27). Substituting the FG expansions (2.39), applicable for both the A_i and A'_i fields, into (6.5), we obtain

$$A'_{(1)i} = \varepsilon_i{}^{jk}\partial_j A_{(0)k}, \quad A_{(1)i} = -\varepsilon_i{}^{jk}\partial_j A'_{(0)k}\,. \tag{6.7}$$

In the Euclidean signature, the first equation implies

$$\frac{\delta S[A']}{\delta A'_{(0)i}}\bigg|_{\omega=0} = {\rm i}\,\varepsilon_{ijk}\partial_j \frac{\delta S[A]}{\delta A_{(1)k}}\bigg|_{\omega=\pi/8}\,. \tag{6.8}$$

In the $\omega = 0$ case, $A'_{(0)i}$ is treated as the source and $A'_{(1)i}$ is the VEV, whereas in the $\omega = \pi/8$ case, the roles of $A_{(0)i}$ and $A_{(1)i}$ interchange, with $A_{(0)i}$ becoming the VEV and $A_{(1)i}$ playing the role of the source. Differentiating (6.8) with respect to $A'_{(0)j}$ and using (6.7), we derive the relation (6.3). In fact, it follows from (6.8) that $(n+m)$-point functions obey, schematically,

$$\langle \mathcal{O}_1\cdots\mathcal{O}_n J_1^{\rm top}\cdots J_m^{\rm top}\rangle\Big|_{\omega=\pi/8} = \langle \mathcal{O}_1\cdots\mathcal{O}_n J_1\cdots J_m\rangle\Big|_{\omega=0}\,, \tag{6.9}$$

where the $\mathcal{O}$ denote scalar, vector or tensor primary operators in the ABJM model.

As a non-trivial check of this relation, we now examine the 3-point correlation functions calculated in the previous section. In the $\omega = \pi/8$ deformed theory, we found

$$\begin{aligned}
\langle \mathcal{O}^{\Delta=1}(\vec{x}_1)^{IJ} J_i(\vec{x}_2)^{KL} A_j(\vec{x}_3)\rangle\Big|_{\omega=\pi/8} &= -T^{(1)IJ,KL}_{ij,\Delta=1}\,,\\
\langle \mathcal{O}^{\Delta=2}(\vec{x}_1)^{IJ} J_i(\vec{x}_2)^{KL} A_j(\vec{x}_3)\rangle\Big|_{\omega=\pi/8} &= T^{(2)IJ,KL}_{ij,\Delta=2}\,.
\end{aligned} \tag{6.10}$$

In the $\omega = 0$ theory, the cubic-interaction vertices (5.1) take the form

$$\begin{aligned}
&1) \qquad -\sqrt{2}\int \frac{d^4z}{z_0^4}\mathcal{S}^{IJ}\partial_{[\mu}A_{\nu]}{}^{IJ}\,\partial_\mu A_\nu\,,\\
&2) \qquad -\frac{\rm i}{\sqrt{2}}\int \frac{d^4z}{z_0^4}\mathcal{P}^{IJ}\,\epsilon_{\mu\nu\rho\sigma}\partial_\mu A_\nu{}^{IJ}\partial_\rho A_\sigma\,.
\end{aligned} \tag{6.11}$$

Unlike the $\omega = \pi/8$ case, when $\omega = 0$ the U(1) gauge field satisfies a Dirichlet boundary condition, and therefore the dual of the bulk U(1) gauge field is a conserved spin-1 current.

According to the AdS/CFT dictionary, the 3-point correlation function computed in the $\omega = 0$ theory corresponds to a 3-point function involving two conserved spin-1 currents and a scalar operator. Similar calculations lead to the 3-point correlation functions associated with the first and second cubic vertices, which are given by

$$\begin{aligned}\langle \mathcal{O}^{\Delta=2}(\vec{x}_1)^{IJ} J_i(\vec{x}_2)^{KL} J_j(\vec{x}_3)\rangle\Big|_{\omega=0} &= \widetilde{T}^{(1)IJ,KL}_{ij,\Delta=2}\,,\\ \langle \mathcal{O}^{\Delta=1}(\vec{x}_1)^{IJ} J_i(\vec{x}_2)^{KL} J_j(\vec{x}_3)\rangle\Big|_{\omega=0} &= \widetilde{T}^{(2)IJ,KL}_{ij,\Delta=1}\,,\end{aligned} \tag{6.12}$$

where

$$\begin{aligned}\widetilde{T}^{(1)IJ,KL}_{ij}(\vec{x}_1,\vec{x}_2,\vec{x}_3) = &-\frac{\Gamma[\frac{1+\Delta}{2}]\Gamma[\frac{\Delta}{2}]\Gamma[\frac{\Delta}{2}+1]\Gamma[2-\frac{\Delta}{2}]}{8\sqrt{2}\pi^4\Gamma[\Delta-\frac{3}{2}]}(\delta^{IK}\delta^{JL}-\delta^{IL}\delta^{JK})\\ &\times\frac{(\Delta-4)J_{ij}(\vec{x}_2-\vec{x}_3)+\Delta J_{ik}(\vec{x}_2-\vec{x}_1)J_{kj}(\vec{x}_1-\vec{x}_3)}{|\vec{x}_2-\vec{x}_3|^{4-\Delta}|\vec{x}_1-\vec{x}_3|^{\Delta}|\vec{x}_1-\vec{x}_2|^{\Delta}}\,,\end{aligned} \tag{6.13}$$

$$\begin{aligned}\widetilde{T}^{(2)IJ,KL}_{ij}(\vec{x}_1,\vec{x}_2,\vec{x}_3) = &-\mathrm{i}\,\frac{\Gamma[\frac{1+\Delta}{2}]^2\Gamma[\frac{\Delta}{2}+1]\Gamma[\frac{5-\Delta}{2}]}{2\sqrt{2}\pi^4\Gamma[\Delta-\frac{3}{2}]}(\delta^{IK}\delta^{JL}-\delta^{IL}\delta^{JK})\\ &\times\frac{|\vec{x}_1-\vec{x}_3|^{1-\Delta}\varepsilon_{mkj}}{|\vec{x}_1-\vec{x}_2|^{\Delta+1}|\vec{x}_2-\vec{x}_3|^{3-\Delta}}J_{ki}(\vec{x}_2-\vec{x}_3)\left[\frac{(x_3-x_1)_m}{|\vec{x}_3-\vec{x}_1|^2}-\frac{(x_3-x_2)_m}{|\vec{x}_3-\vec{x}_2|^2}\right].\end{aligned} \tag{6.14}$$

The form of our 3-point correlation function matches with the general structure of $\langle \mathcal{O}JJ\rangle$ obtained from a CFT calculation by utilizing conformal symmetry and current conservation [27].

Using the lemmata

$$\begin{aligned}\mathrm{i}\,\varepsilon_{jkm}\frac{\partial}{\partial x_{3k}}T^{(1)IJ,KL}_{im,\Delta=1}(\vec{x}_1,\vec{x}_2,\vec{x}_3) &= -\widetilde{T}^{(2)IJ,KL}_{ij,\Delta=1}(\vec{x}_1,\vec{x}_2,\vec{x}_3)\,,\\ \mathrm{i}\,\varepsilon_{jkm}\frac{\partial}{\partial x_{3k}}T^{(2)IJ,KL}_{im,\Delta=2}(\vec{x}_1,\vec{x}_2,\vec{x}_3) &= \widetilde{T}^{(1)IJ,KL}_{ij,\Delta=2}(\vec{x}_1,\vec{x}_2,\vec{x}_3)\,,\end{aligned} \tag{6.15}$$

we find that

$$\langle \mathcal{O}^{\Delta=1,2}(\vec{x}_1)^{IJ} J_i(\vec{x}_2)^{KL} J^{\rm top}_j(\vec{x}_3)\rangle\Big|_{\omega=\pi/8} = \langle \mathcal{O}^{\Delta=1,2}(\vec{x}_1)^{IJ} J_i(\vec{x}_2)^{KL} J_j(\vec{x}_3)\rangle\Big|_{\omega=0}\,, \tag{6.16}$$

which agrees with (6.9).

In seeking a holographic CFT interpretation of these results, we know that the electric-magnetic duality of the U(1) gauge field in the bulk can be understood in terms of the called S-transformation of the boundary CFT, in which a global U(1) symmetry is gauged and an off-diagonal Chern-Simons term is added to the CFT [6]. Noting that in addition to the $\mathrm{U}(N)_k\times\mathrm{U}(N)_{-k}$ ABJM model there also exists an $\mathrm{SU}(N)_k\times\mathrm{SU}(N)_{-k}$ ABJM model, it is tempting to interpret the former as the S-transform of the latter. This is motivated by the fact that the bosonic part of the $\mathrm{U}(1)\times\mathrm{U}(1)$ sector of the ABJM model has the form

$$\mathcal{L}_{\rm ABJM} = -\mathrm{tr}|(\partial_i - iB_i)C^I|^2 + \varepsilon^{ijk}A_i\partial_jB_k\,,\quad I = 1\cdots 4\,, \tag{6.17}$$

where C^I are scalar fields in the bi-fundamental representation of $\mathrm{U}(N)\times\mathrm{U}(N)$, and A_i, B_i are defined in terms of the diagonal $\mathrm{U}(1)\times\mathrm{U}(1)$ gauge fields as $B_i = A_{1i} - A_{2i}$ and $A_i = A_{1i} + A_{2i}$. However, it has been pointed out that the bulk dual of the $\mathrm{SU}(N)_k \times \mathrm{SU}(N)_{-k}$ model is not simply related to the undeformed $\mathcal{N} = 6$ theory [28].

Turning to the $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ ABJM model, in addition to the U(1) current whose bosonic part is

$$J_i = \mathrm{i}\,\mathrm{tr}\left(C^{I\star}\overset{\leftrightarrow}{\partial}_i C^I\right) , \tag{6.18}$$

there also exists a topological current given by

$$J_i^{\mathrm{top}} = \varepsilon_{ijk}\partial_j A_k \, . \tag{6.19}$$

The B_i equation implies

$$J_i^{\mathrm{top}} = J_i \, . \tag{6.20}$$

Thus, the relation (6.9) that we found from the bulk point of view is manifestly realized due to the on-shell identification of J_i^{top} with the Noether current J in the $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ ABJM model. Therefore we observe that the holographic dual of the $\omega = \pi/8$ theory is not a new CFT, but instead the ABJM model itself, in the sense that the processes involving the Noether current J and those involving the dynamical U(1) in the ABJM model are described by ostensibly distinct bulk theories with $\omega = 0$ and $\omega = \pi/8$ respectively, which, in turn, are related to each other by electric-magnetic duality.

7 Conclusions

The main goal of this paper has been to initiate an investigation of correlation functions in the conformal field theories holographically dual to the recently discovered ω deformations of gauged supergravities. For simplicity, our principal focus has been on the $\mathcal{N} = 6$ supersymmetric supergravities. However, since we obtained these ω-deformed $\mathcal{N} = 6$ theories by truncation from $\mathcal{N} = 8$, it was of interest to study some of the aspects of the ω deformations also in the full $\mathcal{N} = 8$ gauged supergravities. We therefore also examined the supersymmetry-preserving boundary conditions in ω-deformed SO(8) gauged $\mathcal{N} = 8$ supergravity. The inequivalent such theories are characterized by ω lying in the interval $[0, \pi/8]$. For any non-vanishing ω in this interval we find that consistent Fefferman-Graham boundary conditions for fluctuations around the trivial AdS_4 vacuum can be compatible with at most an $\mathcal{N} = 3$ subset of the $\mathcal{N} = 8$ supersymmetries. This result is obtained under the assumption that the graviton and gravitini must necessarily obey Dirichlet boundary conditions, so that they do not correspond to propagating spin-2 or spin-3/2 modes in the dual boundary theory. In this $\mathcal{N} = 3$ case, all the vectors must also obey Dirichlet boundary conditions. We also find that $\mathcal{N} = 1$ is the maximum allowed supersymmetry for which some of the vectors can instead obey Neumann boundary conditions, when ω is non-vanishing. Furthermore, we established that mixed boundary conditions on the vector fields, where a given vector would have both electric and magnetic components on the boundary, are not allowed for any $\mathcal{N} \geq 1$ supersymmetry.

JHEP02(2015)112

If ω does vanish, $\mathcal{N} = 8$ supersymmetry is allowed with all spin≥ 1 fields obeying Dirichlet boundary conditions [26]. We also found that at $\omega = 0$ a maximum of $\mathcal{N} = 2$ supersymmetry is compatible with Neumann boundary conditions imposed on a subset of the vector fields.

The situation as regards supersymmetry-preserving boundary conditions is very different if we actually truncate the $\mathcal{N} = 8$ theory to a theory with a lower degree of supersymmetry. Among such theories, in this paper we have studied the ω-deformed SO(6) gauged $\mathcal{N} = 6$ supergravity. The undeformed version of this truncation was studied in [5]. We constructed the $\mathcal{N} = 6$ truncation for the ω-deformed theory, and showed that the ω deformation survives, again in the interval $[0, \pi/8]$. We also exhibited the underlying $SO^*(12)/\mathrm{U}(6)$ coset structure of the couplings. Furthermore, we found that $\mathcal{N} = 6$ supersymmetry-preserving boundary conditions are possible, provided that $\omega = \pi/8$ (or $\omega = 0$). When $\omega = \pi/8$, the SO(6) gauge fields still obey Dirichlet boundary conditions, but the additional U(1) gauge field obeys a Neumann boundary condition. The $\omega = 0$ and $\omega = \pi/8$ theories are related by a U(1) electric-magnetic duality.

The U(1) gauge field appears in the equations of motion only through its field strength, since none of the $\mathcal{N} = 6$ fields are charged under the U(1). We showed that the embedding of the ω-deformed $\mathcal{N} = 6$ theories into IIA supergravity reduced on $\mathbb{CP}^3$ can be straightforwardly accomplished, in view of the fact that Kaluza-Klein reductions on spheres or other curved manifolds are necessarily performed at the level of the equations of motion, as explained in more detail in appendix A.

We computed the leading-order examples of ω-dependent tree-level amplitudes in the $\mathcal{N} = 6$ theories, as a step towards understanding the ω deformation from a dual holographic viewpoint. Up to 3-point tree-level graphs, we found that the only ω-dependent amplitudes are those involving the trilinear coupling of a SO(6) gauge field with the U(1) gauge field and a scalar or pseudoscalar field. We computed these amplitudes, which are parity-violating. The amplitudes turned out to be finite without the need for any regularisation. These results would also have a wider applicability in other situations where one has gauge fields obeying Neumann boundary conditions as well as gauge fields obeying Dirichlet boundary conditions.

We also computed the associated correlation functions in the undeformed theory. Inspired by Witten's holographic interpretation of bulk electric-magnetic duality [6], we found that the electric-magnetic duality transformation of the U(1) field that is required when relating the $\omega = 0$ and $\omega = \pi/8$ bulk theories has the effect of interchanging the Noether current and topological current in the amplitudes of the $\mathrm{U}(1) \times \mathrm{U}(1)$ sector of the $\mathrm{U}(N)_k \times \mathrm{U}(N)_{-k}$ ABJM model.

Although we focused on computing the amplitudes in the ω-deformed SO(6) gauged supergravity, it would be interesting to study other ω-deformed gauged supergravities that admit an AdS_4 vacuum, in framework of the AdS/CFT correspondence. At present, the following gauged $\mathcal{N} = 8$ supergravity theories are known to admit ω deformations with supersymmetric AdS vacua [29]:

- SO(8) supergravity with $\mathcal{N} = 8$ supersymmetry.

- SO(1, 7) and [SO(1, 1) × SO(6)] ⋉ T^{12} supergravities with $\mathcal{N} = 4$ supersymmetry.
- SO(8), SO(1, 7) and ISO(1, 7) gauged supergravities with $\mathcal{N} = 3$ supersymmetry.

We have shown in this paper that in the SO(8) gauged $\mathcal{N} = 8$ theory, the boundary conditions preserve at most $\mathcal{N} = 3$ supersymmetry for non-vanishing ω, even though the vacuum itself preserves $\mathcal{N} = 8$ supersymmetry. The supersymmetry-preserving boundary conditions and holographic aspects for the remaining supersymmetric vacua deserve further investigation.

Acknowledgments

We are grateful to Daniel Jafferis, Hermann Nicolai, Gianluca Inverso, Xi Yin for helpful discussions. Y.P. is grateful to the Mainz Institute for Theoretical Physics (MITP) for its hospitality and its partial support during the completion of this work. A.B. is grateful to the Mitchell Institute for hospitality during an extended visit to Texas A&M University. The work of C.N.P. is supported in part by DOE grant DE-FG02-13ER42020. The work of E.S. is supported in part by NSF grant PHY-1214344.

A ω-deformed $\mathcal{N} = 6$ supergravity and higher dimensions

An outstanding problem is to find whether the ω-deformed supergravities have any higher-dimensional origin. In the case of ω-deformed $\mathcal{N} = 8$ supergravity, it has been suggested that to embed it into eleven-dimensional supergravity would probably require first extending the eleven-dimensional theory to some kind of a doubled theory include a "dual graviton" in addition to the usual one [4]. The idea, essentially, is that the ω deformation amounts to a gauging in which some combination of the 28 dual vector gauge fields as well as the 28 original gauge fields of the four-dimensional theory would participate in the minimal couplings to the other fields of the $\mathcal{N} = 8$ multiplet, and such gaugings could not arise unless the dual fields were already themselves embedded into the eleven-dimensional theory, as components of a dual graviton. However, the construction of such a doubled eleven-dimensional theory, with non-linear couplings for the dual gravitons, remains an open problem.

The situation is rather different in the case of the ω-deformed $\mathcal{N} = 6$ supergravities. As we discussed in section 3, at the level of the four-dimensional equations of motion the ω parameter in the deformed $\mathcal{N} = 6$ theories can be absorbed by means of a duality transformation of the U(1) gauge field. This can be done because unlike the SO(6) gauge fields, which have minimal couplings to other fields in supermultiplet, the U(1) gauge field has no minimal couplings, and it enters the equations of motion purely through its field strength. The non-triviality of the ω parameter in the $\mathcal{N} = 6$ theory stems solely from the fact that it cannot be absorbed by any local field redefinition at the level of the Lagrangian, and thus it can affect quantum properties of the theory (such as correlation functions in the dual theory).

For the above reasons, the question of whether the ω-deformed $\mathcal{N} = 6$ supergravities can be embedded in a higher-dimensional theory is rather different from the $\mathcal{N} = 8$ case.

First of all, we note that the highly non-trivial mechanism whereby a sphere reduction can give rise to a consistent truncation in the lower dimension is one that always operates at the level of the equations of motion, rather than at the level of the Lagrangian. In other words, the lower-dimensional theory, such as the standard $\mathcal{N}=8$ gauged supergravity, emerges at the level of the equations of motion when the reduction ansatz is substituted into the higher-dimensional equations of motion. One cannot instead substitute into the higher-dimensional action and thereby obtain the lower-dimensional action. An illustration, pertinent to our present discussion, of why this is the case is provided by a very early example of a non-trivial consistent sphere reduction that was obtained in [30]. In that paper it was shown that the four-dimensional Einstein-Maxwell theory with a negative cosmological constant could be consistently embedded in eleven-dimensional supergravity, whose bosonic Lagrangian is $\mathcal{L}_{11} = \hat{R}\,\hat{*}\mathbb{1} - \frac{1}{2}\hat{*}\hat{F}_{(4)} \wedge \hat{F}_{(4)} + \frac{1}{6}\hat{F}_{(4)} \wedge \hat{F}_{(4)} \wedge \hat{A}_{(3)}$, with the reduction ansatz being given by [30]

$$\begin{aligned} d\hat{s}_{11}^2 &= ds_4^2 + (d\psi + A + B)^2 + d\Sigma_6^2\,, \\ \hat{F}_{(4)} &= 6m\epsilon_{(4)} - *F \wedge J\,. \end{aligned} \tag{A.1}$$

Here ds_4^2 is the four-dimensional metric, with volume form $\epsilon_{(4)}$, $d\Sigma_6^2$ is the Fubini-Study metric on $\mathbb{CP}^3$ with Ricci tensor $R_{ab} = 8m^2 g_{ab}$, $dB = 2mJ$ where J is the Kähler form of $\mathbb{CP}^3$, and $*$ denotes the four-dimensional Hodge dual. This reduction ansatz obeys the eleven-dimensional equations of motion, and the Bianchi identity $d\hat{F}_{(4)} = 0$, if and only if the four-dimensional fields $g_{\mu\nu}$ and A_μ satisfy the Einstein-Maxwell equations [30]

$$R_{\mu\nu} = 2\Big(F_{\mu\rho}F_\nu{}^\rho - \frac{1}{4}F^2\, g_{\mu\nu}\Big) - 12m^2 g_{\mu\nu}\,, \qquad \nabla_\mu F^{\mu\nu} = 0\,. \tag{A.2}$$

The fact that the ansatz for the eleven-dimensional 4-form in (A.1) obeys the Bianchi identity $d\hat{F}_{(4)} = 0$ only upon the use of the four-dimensional equations of motion illustrates the fact that one could not write down a reduction ansatz on the original fundamental fields $\hat{g}_{MN}$ and $\hat{A}_{(3)}$ appearing in the eleven-dimensional Lagrangian.

The embedding of the Einstein-Maxwell theory given by (A.1) is in fact itself a consistent truncation of the embedding of the (bosonic sector) of the standard gauged $\mathcal{N}=6$ supergravity into eleven dimensions, namely where the fields are truncated to the SO(6) singlets. The Maxwell field in (A.1) is precisely the U(1) gauge field of the $\mathcal{N}=6$ theory.

The question of whether one can embed the ω-deformed family of $\mathcal{N}=6$ gauged supergravities in eleven-dimensional supergravity is a slightly tricky one. Since the consistent reduction must be performed at the level of the equations of motion, and since the ω parameter in the gauged $\mathcal{N}=6$ supergravities can be absorbed by means of local scalar field redefinitions and a U(1) duality transformation at the level of the equations of motion, it follows that the entire family of ω-deformed theories can be embedded into eleven-dimensional supergravity. Of course, since the U(1) gauge potentials for two different values of ω are non-locally related, and since the bare U(1) gauge potential appears in the metric ansatz, as in the further truncation in (A.1), this means that the eleven-dimensional embeddings for two different values of ω would be non-locally related.

One can instead consider the embedding the ω-deformed $\mathcal{N}=6$ supergravities into the ten-dimensional type IIA supergravity. As was shown in [32], if one makes an S^1 reduction on the Hopf fibres of the S^7 embedding of $\mathcal{N}=8$ supergravity into $D=11$, truncating to the U(1) singlets, one obtains the $\mathcal{N}=6$ gauged supergravity as a consistent reduction of type IIA supergravity on $\mathbb{CP}^3$. In particular, the U(1) gauge field in the $\mathcal{N}=6$ theory is now coming not from the ten-dimensional metric, but rather, it is the Ramond-Ramond 2-form field strength already present in the type IIA theory. (Which arose, of course, from the Kaluza-Klein vector of the S^1 reduction from $D=11$.) Thus in the embedding of the $\mathcal{N}=6$ theory into the type IIA theory the reduction ansatz only requires the knowledge of the U(1) *field strength*, and not its 1-form potential. Accordingly, not only can one embed any of the ω-deformed $\mathcal{N}=6$ supergravities into ten-dimensional type IIA supergravity, but also the relation between the embeddings for two different values of ω can now be expressed purely locally, since the U(1) gauge potential does not appear in the reduction ansatz from $D=10$ to $D=4$.

B Supersymmetric boundary conditions in ω-deformed $\mathcal{N}=8$ theory

It was shown long ago that in the de Wit-Nicolai theory, there exist $\mathcal{N}=8$ supersymmetry-preserving boundary conditions [26]. Explicitly, these boundary conditions, with the gauge choices (2.38), are given by[3]

$$e^{\hat{r}}_{(0)i}=\delta^{\hat{r}}_i\,,\quad \psi^I_{(0)i}=0\,,\quad A^{IJ}_{(0)i}=0\,,\quad \chi^{IJK}=0\,,\quad \mathcal{S}^{IJKL}_{(2)}=0\,,\quad \mathcal{P}^{IJKL}_{(1)}=0\,. \tag{B.1}$$

In this appendix, we shall show that in the ω-deformed $\mathcal{N}=8$ theories one can no longer impose consistent boundary conditions that preserve the full $\mathcal{N}=8$ supersymmetry. Note that we are making the key assumption in this analysis that the fields of spins 2 and 3/2 must obey Dirichlet boundary conditions, to avoid having propagating graviton or gravitino modes in the boundary theory. The goal in this appendix is then to determine the possible numbers of supersymmetries that *are* preserved by appropriate choices of boundary conditions in the full ω-deformed $\mathcal{N}=8$ theory.

Because shall be considering a subset $\mathcal{N}<8$ of the full $\mathcal{N}=8$ supersymmetry of the theory itself, the supersymmetry variations of the boundary conditions on the gravitini will only determine the boundary conditions on a subset of the vector fields. For the remaining vector fields, we shall begin by considering the cases of purely Dirichlet or Neumann boundary conditions on these fields. Later on, when considering the $\mathcal{N}=1$ case, we shall allow for all possible mixed boundary conditions on the vector fields. We shall show, however, that such mixed boundary conditions are ruled out. Applying similar considerations, we then show that mixed boundary conditions on vectors are not possible for any $\mathcal{N}>1$ either, thus justifying our previous restriction to the purely Dirichlet or Neumann possibilities.

[3]In this appendix we shall use $I, J, \ldots$ to denote SO(8) indices, while $i, j, \ldots$ denote boundary coordinate indices.

B.1 $\mathcal{N} \geq 4$ supersymmetry

We shall first show that in the ω-deformed $\mathcal{N} = 8$ theory there are no boundary conditions that preserve an $\mathcal{N} = 4$ subset of the $\mathcal{N} = 8$ supersymmetries. In turn, this then implies that there cannot be boundary conditions preserving any number $\mathcal{N} \geq 4$ of the supersymmetries, either.

To show that $\mathcal{N} = 4$ supersymmetry-preserving boundary conditions are not possible when $\omega \neq 0$, we split the SO(8) indices into $I, J = 1, 2, 3, 4$, and $r, s = 5, \dots 8$. It suffices in this $\mathcal{N} = 4$ case to consider the boundary conditions on the vector fields A_i^{rs}. Consider first the case when they obey Dirichlet boundary conditions, i.e. $A_{(0)i}^{rs} = 0$. The supersymmetry variation of this boundary condition, under the $\mathcal{N} = 4$ subset ϵ^I of the supersymmetry parameters (i.e. with $\epsilon^r = 0$), leads to

$$\cos 2\omega\, \mathcal{P}_{(1)}^{rsIJ} + \sin 2\omega\, \mathcal{S}_{(1)}^{rsIJ} = 0, \quad \sin 2\omega\, \mathcal{P}_{(2)}^{rsIJ} - \cos 2\omega\, \mathcal{S}_{(2)}^{rsIJ} = 0\,. \tag{B.2}$$

It then follows from the $SU8$) duality condition (2.37) that

$$\cos 2\omega\, \mathcal{P}_{(1)}^{rsIJ} - \sin 2\omega\, \mathcal{S}_{(1)}^{rsIJ} = 0, \quad \sin 2\omega\, \mathcal{P}_{(2)}^{rsIJ} + \cos 2\omega\, \mathcal{S}_{(2)}^{rsIJ} = 0\,. \tag{B.3}$$

Therefore, if $\omega \neq 0$, then we find $\mathcal{S}_{(1)}^{rsIJ} = \mathcal{P}_{(1)}^{rsIJ} = \mathcal{S}_{(2)}^{rsIJ} = \mathcal{P}_{(2)}^{rsIJ} = 0$, which are not acceptable as boundary conditions since they would imply that these scalars all vanished everywhere.

Now consider instead imposing Neumann boundary conditions on A_i^{rs}. If these boundary conditions preserved $\mathcal{N} = 4$ supersymmetry, they would also imply the existence of such boundary conditions that preserved $\mathcal{N} = 3$ supersymmetry. However, as we shall show explicitly in the next subsection, there cannot exist $\mathcal{N} = 3$ boundary conditions with A_i^{rs} satisfying Neumann boundary condition. Thus we have established that when $\omega \neq 0$ there can exist no choice of boundary conditions that preserves $\mathcal{N} \geq 4$ supersymmetry.

B.2 $\mathcal{N} = 3$ supersymmetry

In the $\mathcal{N} = 3$ case, we decompose the SO(8) indices so that $I, J = 1, 2, 3$, and $r, s = 4, 5, \dots 8$. As we shall see below, there do in fact exist consistent $\mathcal{N} = 3$ boundary conditions for $\omega \neq 0$, and, in particular, we can no longer derive an immediate contradiction, as we did for $\mathcal{N} = 4$, merely by considering the restrictions following from imposing Dirichlet or Neumann boundary conditions on A_i^{rs}. Instead, we begin the analysis here by noting that the supersymmetry variation of the Dirichlet boundary condition on the gravitini requires

$$A_{(0)i}^{rI} = 0, \qquad A_{(0)i}^{IJ} = 0\,. \tag{B.4}$$

The vanishing of the supersymmetry variations of $A_{(0)i}^{rI}$ and $A_{(0)i}^{IJ}$ then imply

$$\cos 2\omega\, \chi_+^{rIJ} + \mathrm{i}\, \sin 2\omega \gamma_5 \chi_-^{rIJ} = 0, \quad \cos 2\omega\, \chi_+^{IJK} + \mathrm{i}\, \sin 2\omega\, \gamma_5 \chi_-^{IJK} = 0\,. \tag{B.5}$$

The second equation is automatically invariant under $\mathcal{N} = 3$ supersymmetry variations. Demanding that the first equation be invariant under

$$\delta \chi_+^{rIJ} = -\mathcal{S}_{(2)}^{rIJK} \epsilon_+^K + 2\mathrm{i}\, \mathcal{P}_{(1)}^{rstI} \gamma_5 \epsilon_-^I - \mathrm{i} \not{D} \mathcal{P}_{(1)}^{rIJK} \gamma_5 \epsilon_+^K\,,$$

$$\delta\chi_-^{rIJ} = 2\mathcal{S}_{(1)}^{rIJK}\epsilon_-^K - \mathrm{i}\mathcal{P}_{(2)}^{rIJK}\gamma_5\epsilon_+^K + \not{D}\mathcal{S}_{(1)}^{rIJK}\epsilon_+^K\,, \tag{B.6}$$

leads to

$$\cos 2\omega\ \mathcal{P}_{(1)}^{rIJK} + \sin 2\omega\ \mathcal{S}_{(1)}^{rIJK} = 0, \quad \sin 2\omega\ \mathcal{P}_{(2)}^{rIJK} - \cos 2\omega\ \mathcal{S}_{(2)}^{rIJK} = 0\,. \tag{B.7}$$

Using the duality property (2.37) of the scalars, the above equations imply that

$$\cos 2\omega\ \mathcal{P}_{(1)}^{rstp} - \sin 2\omega\ \mathcal{S}_{(1)}^{rstp} = 0, \quad \sin 2\omega\ \mathcal{P}_{(2)}^{rstp} + \cos 2\omega\ \mathcal{S}_{(2)}^{rstp} = 0\,. \tag{B.8}$$

It can be verified that the first equation in (B.5) guarantees that equations (B.7) and (B.8) are invariant under the $\mathcal{N} = 3$ supersymmetry variations.

We now consider the possible boundary conditions on the vector fields A_i^{rs}. Let us first consider imposing Dirichlet boundary conditions on A_i^{rs}. The vanishing of the variation of $A_{(0)i}^{rs}$ requires

$$\cos 2\omega\ \chi_+^{rsI} + \mathrm{i}\ \sin 2\omega\ \gamma_5\chi_-^{rsI} = 0\,. \tag{B.9}$$

The supersymmetry variation of this condition, using

$$\begin{aligned}
\delta\chi_+^{rsI} &= -\mathcal{S}_{(2)}^{rsIJ}\epsilon_+^J + 2\mathrm{i}\,\mathcal{P}_{(1)}^{rsIJ}\gamma_5\epsilon_-^J - \mathrm{i}\not{D}\mathcal{P}_{(1)}^{rsIJ}\gamma_5\epsilon_+^J\,,\\
\delta\chi_-^{rsJ} &= 2\mathcal{S}_{(1)}^{rsIJ}\epsilon_-^J - \mathrm{i}\mathcal{P}_{(2)}^{rsIJ}\gamma_5\epsilon_+^J + \not{D}\mathcal{S}_{(1)}^{rsIJ}\epsilon_+^J\,,
\end{aligned} \tag{B.10}$$

implies

$$\cos 2\omega\ \mathcal{P}_{(1)}^{rsIJ} + \sin 2\omega\ \mathcal{S}_{(1)}^{rsIJ} = 0, \quad \sin 2\omega\ \mathcal{P}_{(2)}^{rsIJ} - \cos 2\omega\ \mathcal{S}_{(2)}^{rsIJ} = 0\,. \tag{B.11}$$

Using the duality property (2.37) of the scalars, (B.11) implies

$$\cos 2\omega\ \mathcal{P}_{(1)}^{Irst} - \sin 2\omega\ \mathcal{S}_{(1)}^{Irst} = 0, \quad \sin 2\omega\ \mathcal{P}_{(2)}^{Irst} + \cos 2\omega\ \mathcal{S}_{(2)}^{Irst} = 0\,. \tag{B.12}$$

To preserve these conditions under the $\mathcal{N} = 3$ supersymmetry variations

$$\begin{aligned}
\delta\mathcal{S}_{(1)}^{Irst} &= \bar{\epsilon}_+^I\chi_-^{rst} - \frac{1}{2}\varepsilon^{IJK}\epsilon^{rstpq}\bar{\epsilon}_+^J\chi_-^{Kpq}\,,\\
\delta\mathcal{P}_{(1)}^{Irst} &= -\mathrm{i}\left(\bar{\epsilon}_+^I\gamma_5\chi_+^{rst} + \frac{1}{2}\varepsilon^{IJK}\epsilon^{rstpq}\bar{\epsilon}_+^J\gamma_5\widetilde{\chi}_+^{Kpq}\right),\\
\delta\mathcal{S}_{(2)}^{Irst} &= \left(\bar{\epsilon}_-^I\chi_+^{rst} - \frac{1}{2}\varepsilon^{IJK}\epsilon^{rstpq}\bar{\epsilon}_-^J\chi_+^{Kpq} + \bar{\epsilon}_+^I\not{D}\chi_+^{rst} - \frac{1}{2}\varepsilon^{IJK}\epsilon^{rstpq}\bar{\epsilon}_+^J\not{D}\chi_+^{Kpq}\right),\\
\delta\mathcal{P}_{(2)}^{Irst} &= -\mathrm{i}\left(\bar{\epsilon}_-^I\gamma_5\chi_-^{rst} + \frac{1}{2}\varepsilon^{IJK}\epsilon^{rstpq}\bar{\epsilon}_-^J\gamma_5\chi_-^{Kpq} - \bar{\epsilon}_+^I\gamma_5\not{D}\chi_-^{rst]} - \frac{1}{2}\varepsilon^{IJK}\epsilon^{rstpq}\bar{\epsilon}_+^J\gamma_5\not{D}\chi_-^{Kpq}\right),
\end{aligned} \tag{B.13}$$

we need to impose

$$\cos 2\omega\ \chi_+^{rst} - \mathrm{i}\ \sin 2\omega\ \gamma_5\chi_-^{rst} = 0\,. \tag{B.14}$$

The condition (B.14) is preserved under the supersymmetry variations

$$\delta\chi_+^{rst} = -\mathcal{S}_{(2)}^{rstI}\epsilon_+^I + 2\mathrm{i}\,\mathcal{P}_{(1)}^{rstI}\gamma_5\epsilon_-^I - \mathrm{i}\not{D}\mathcal{P}_{(1)}^{rstI}\gamma_5\epsilon_+^I\,,$$

$$\delta\chi_-^{rst} = 2\mathcal{S}_{(1)}^{rstI}\epsilon_-^I - \mathrm{i}\mathcal{P}_{(2)}^{rstI}\gamma_5\epsilon_+^I + \not{D}\mathcal{S}_{(1)}^{rstI}\epsilon_+^I\,, \tag{B.15}$$

as a consequence of (B.12).

In summary, we have found a consistent set of $\mathcal{N}=3$ supersymmetry-preserving boundary conditions, which takes the form

$$\begin{aligned}
A^{rs}_{(0)i} = 0, \qquad A^{rI}_{(0)i} &= 0, & A^{IJ}_{(0)} &= 0\,,\\
\cos 2\omega\ \chi_+^{rIJ} + \mathrm{i}\sin 2\omega\ \gamma_5\chi_-^{rIJ} &= 0, & \cos 2\omega\ \chi_+^{IJK} + \mathrm{i}\sin 2\omega\ \gamma_5\chi_-^{IJK} &= 0\,,\\
\cos 2\omega\ \chi_+^{rsI} + \mathrm{i}\sin 2\omega\ \gamma_5\chi_-^{rsI} &= 0, & \cos 2\omega\ \chi_+^{rst} - \mathrm{i}\sin 2\omega\ \gamma_5\chi_-^{rst} &= 0\,,\\
\cos 2\omega\ \mathcal{P}_{(1)}^{Irst} - \sin 2\omega\ \mathcal{S}_{(1)}^{Irst} &= 0\,, & \sin 2\omega\ \mathcal{P}_{(2)}^{Irst} + \cos 2\omega\ \mathcal{S}_{(2)}^{Irst} &= 0\,,\\
\cos 2\omega\ \mathcal{P}_{(1)}^{rIJK} + \sin 2\omega\ \mathcal{S}_{(1)}^{rIJK} &= 0\,, & \sin 2\omega\ \mathcal{P}_{(2)}^{rIJK} - \cos 2\omega\ \mathcal{S}_{(2)}^{rIJK} &= 0\,.
\end{aligned} \tag{B.16}$$

Equations (B.8) and (B.11) are implied by the duals of the last two equations in (B.16).

We shall now show that it is not possible to impose instead Neumann boundary conditions on A_i^{rs} while preserving $\mathcal{N}=3$ supersymmetry. Imposing the Neumann boundary condition $A^{rs}_{(1)i}=0$, its supersymmetry variation under

$$\begin{aligned}
\delta A^{rs}_{(1)i} = -\Big(&\mathcal{S}_{(1)}^{rspq}\bar\epsilon_+^I\gamma_{(0)i}\chi_+^{pqI} + \mathrm{i}\,\mathcal{P}_{(1)}^{rspq}\bar\epsilon_+^I\gamma_{(0)i}\gamma_5\chi_-^{pqI}\\
&+\mathcal{S}_{(1)}^{rsIJ}\bar\epsilon_+^K\gamma_{(0)i}\chi_+^{IJK} + \mathrm{i}\,\mathcal{P}_{(1)}^{rsIJ}\bar\epsilon_+^K\gamma_{(0)i}\gamma_5\chi_-^{IJK}\\
&+2\mathcal{S}_{(1)}^{rstI}\bar\epsilon_+^J\gamma_{(0)i}\chi_+^{tIJ} + \mathrm{i}\,2\mathcal{P}_{(1)}^{rstI}\bar\epsilon_+^J\gamma_{(0)i}\gamma_5\chi_-^{tIJ}\\
&-D_i(A_{(0)})(\cos 2\omega\ \bar\epsilon_+^I\chi_-^{rsI} + \mathrm{i}\sin 2\omega\ \bar\epsilon_+^I\gamma_5\chi_+^{rsI})\Big)\,,
\end{aligned} \tag{B.17}$$

requires

$$\sin 2\omega\ \chi_+^{rsI} - \mathrm{i}\cos 2\omega\ \gamma_5\chi_-^{rsI} = 0\,. \tag{B.18}$$

Furthermore, utilizing (B.8), one can see that the vanishing of $A^{rs}_{(1)i}$ also requires

$$\cos 2\omega\ \chi_+^{rsI} + \mathrm{i}\sin 2\omega\ \gamma_5\chi_-^{rsI} = 0\,. \tag{B.19}$$

The above equation, together with the second equation in (B.18), leads to

$$\chi_+^{rsI} = 0, \quad \chi_-^{rsI} = 0\,. \tag{B.20}$$

This is too strong a condition on the fermions χ^{rsI}, since it implies that they vanish everywhere. Thus, we find that there cannot exist any consistent $\mathcal{N}=3$ boundary condition in which A_i^{rs} satisfies Neumann boundary conditions.

B.3 $\mathcal{N}=2$ supersymmetry

There can clearly exist $\mathcal{N}=2$ boundary conditions that simply follow as a reduction of the $\mathcal{N}=3$ boundary conditions that we obtained above. However, since $\mathcal{N}=2$ supersymmetry is less restrictive than $\mathcal{N}=3$, there could also exist further possible boundary conditions that are compatible with $\mathcal{N}=2$ but not with $\mathcal{N}=3$. We shall therefore now proceed to the analyze the case with $\mathcal{N}=2$ supersymmetry. We introduce indices $I,J=1,2$, and let r,s range from 3 to 7. The Dirichlet boundary conditions for the gravitini imply

$$\begin{aligned}
&\psi^I_{(0)i}=0 \quad \Rightarrow \quad A^{IJ}_{(0)i}=0,\\
&\psi^r_{(0)i}=0 \quad \Rightarrow \quad A^{rI}_{(0)i}=0 \qquad \Rightarrow \quad \cos 2\omega\ \chi^{rIJ}_+ + \mathrm{i}\sin 2\omega\ \gamma_5\chi^{rIJ}_- = 0\,.
\end{aligned} \tag{B.21}$$

We now follow a sequence of steps paralleling those that we used for the $\mathcal{N}=3$ case. Using the supersymmetry variations of the leading terms in the Fefferman-Graham expansions for the spin-1, spin-$\frac{1}{2}$ and spin-0 fields, namely

$$\begin{aligned}
\delta\chi^{rIJ}_+ &= 0, \quad \delta\chi^{rIJ}_- = 0\,,\\
\delta\chi^{rsI}_+ &= -\mathcal{S}^{rsIJ}_{(2)}\epsilon^J_+ + 2\mathrm{i}\mathcal{P}^{rsIJ}_{(1)}\gamma_5\epsilon^J_- - \mathrm{i}\not{D}\mathcal{P}^{rsIJ}_{(1)}\gamma_5\epsilon^J_+\,,\\
\delta\chi^{rsI}_- &= 2\mathcal{S}^{rsIJ}_{(1)}\epsilon^J_- - \mathrm{i}\mathcal{P}^{rsIJ}_{(2)}\gamma_5\epsilon^J_+ + \not{D}\mathcal{S}^{rsIJ}_{(1)}\epsilon^J_+\,,\\
\delta\chi^{rst}_+ &= -\mathcal{S}^{rstI}_{(2)}\epsilon^I_+ + 2\mathrm{i}\mathcal{P}^{rstI}_{(1)}\gamma_5\epsilon^I_- - \mathrm{i}\not{D}\mathcal{P}^{rstI}_{(1)}\gamma_5\epsilon^I_+\,,\\
\delta\chi^{rst}_- &= 2\mathcal{S}^{rstI}_{(1)}\epsilon^I_- - \mathrm{i}\mathcal{P}^{rstI}_{(2)}\gamma_5\epsilon^I_+ + \not{D}\mathcal{S}^{rstI}_{(1)}\epsilon^I_+\,,\\
\delta\mathcal{S}^{Irst}_{(1)} &= \left(\bar\epsilon^I_+\chi^{rst}_- - \frac{1}{3!}\varepsilon^{rstpqu}\epsilon^{IJ}\bar\epsilon^J_+\chi^{pqu}_-\right),\\
\delta\mathcal{P}^{Irst}_{(1)} &= -\mathrm{i}\left(\bar\epsilon^I_+\gamma_5\chi^{rst}_+ + \frac{1}{3!}\varepsilon^{rstpqu}\epsilon^{IJ}\bar\epsilon^J_+\gamma_5\chi^{pqu}_+\right),\\
\delta\mathcal{S}^{Irst}_{(2)} &= \left(\bar\epsilon^I_-\chi^{rst}_+ - \frac{1}{3!}\varepsilon^{rstpqu}\epsilon^{IJ}\bar\epsilon^J_-\chi^{pqu}_+\right.\\
&\qquad \left. +\bar\epsilon^I_+\not{D}\chi^{rst}_+ - \frac{1}{3!}\varepsilon^{rstpqu}\epsilon^{IJ}\bar\epsilon^J_+\not{D}\chi^{pqu}_+\right),\\
\delta\mathcal{P}^{Irst}_{(2)} &= -\mathrm{i}\left(\bar\epsilon^I_-\gamma_5\chi^{rst}_- + \frac{1}{3!}\varepsilon^{rstpqu}\epsilon^{IJ}\bar\epsilon^J_-\gamma_5\chi^{pqu}_-\right.\\
&\qquad \left. -\bar\epsilon^I_+\gamma_5\not{D}\chi^{rst]}_- - \frac{1}{3!}\varepsilon^{rstpqu}\epsilon^{IJ}\bar\epsilon^J_+\gamma_5\not{D}\chi^{pqu}_-\right),\\
\delta\mathcal{S}^{IJrs}_{(1)} &= 2\bar\epsilon^{[I}_+\chi^{J]rs}_-, \quad \delta\mathcal{P}^{IJrs}_{(1)} = -2\mathrm{i}\,\bar\epsilon^{[I}_+\gamma_5\chi^{J]rs}_+\,,\\
\delta\mathcal{S}^{IJrs}_{(2)} &= 2\left(\bar\epsilon^{[I}_-\chi^{J]rs}_+ + \bar\epsilon^{[I}_+\not{D}\chi^{J]rs}_+\right),\\
\delta\mathcal{P}^{IJrs}_{(2)} &= -2\mathrm{i}\left(\bar\epsilon^{[I}_-\gamma_5\chi^{J]rs}_- - \bar\epsilon^{[I}_+\gamma_5\not{D}\chi^{J]rs}_-\right),\\
\delta A^{rs}_{(0)i} &= -\left(\cos 2\omega\ \epsilon^I_+\gamma_{(0)i}\chi^{rsI}_+ + \mathrm{i}\sin 2\omega\ \epsilon^I_+\gamma_{(0)i}\gamma_5\chi^{rsI}_-\right),\\
\delta A^{rs}_{(1)i} &= -\left(\mathcal{S}^{rstp}_{(1)}\epsilon^I_+\gamma_{(0)i}\chi^{tpI}_+ + \mathrm{i}\,\mathcal{P}^{rstp}_{(1)}\epsilon^I_+\gamma_{(0)i}\gamma_5\chi^{tpI}_-\right.\\
&\qquad +2\mathcal{S}^{rstI}_{(1)}\epsilon^J_+\gamma_{(0)i}\chi^{tIJ}_+ + 2\mathrm{i}\,\mathcal{P}^{rstI}_{(1)}\epsilon^J_+\gamma_{(0)i}\gamma_5\chi^{tIJ}_-
\end{aligned}$$

$$-D_i(A_{(0)})(\cos 2\omega\ \epsilon_+^I \chi_-^{rsI} + \mathrm{i} \sin 2\omega\ \epsilon_+^I \gamma_5 \chi_+^{rsI})\Big)\,, \tag{B.22}$$

we find that we can obtain consistent $\mathcal{N} = 2$ boundary conditions in which all the spin-1 fields satisfy Dirichlet boundary conditions. The full set of boundary conditions in this case is given by

$$\begin{aligned}
A^{rI}_{(0)i} = 0, \quad A^{IJ}_{(0)i} = 0, \quad A^{rs}_{(0)i} = 0, &\qquad \alpha_{rst}\chi_+^{rst} + \mathrm{i}\,\gamma_5\beta_{rst}\chi_-^{rst} = 0,\\
\cos 2\omega\ \ \chi_+^{rIJ} + \mathrm{i}\ \sin 2\omega\ \ \gamma_5\chi_-^{rIJ} = 0, &\qquad \cos 2\omega\ \chi_+^{rsI} + \mathrm{i}\ \sin 2\omega\ \gamma_5\chi_-^{rsI} = 0\,,\\
\sin 2\omega\ \ \mathcal{S}^{rsIJ}_{(1)} + \cos 2\omega\ \ \mathcal{P}^{rsIJ}_{(1)} = 0\,, &\qquad \cos 2\omega\ \ \mathcal{S}^{rsIJ}_{(2)} - \sin 2\omega\ \ \mathcal{P}^{rsIJ}_{(2)} = 0\,,\\
\beta_{rst}\mathcal{S}^{rstI}_{(1)} + \alpha_{rst}\mathcal{P}^{rstI}_{(1)} = 0\,, &\qquad \alpha_{rst}\mathcal{S}^{rstI}_{(2)} - \beta_{rst}\mathcal{P}^{rstI}_{(2)} = 0,
\end{aligned} \tag{B.23}$$

where the indices r, s, t are not summed, and the coefficients α_{rst} and β_{rst} are constants. The point here is that the leading-order terms in the Fefferman-Graham expansions of the fermions χ^{rst}, together with those of the scalars $\mathcal{S}^{rstI}$ and $\mathcal{P}^{rstI}$, form a closed multiplet, whose supersymmetry variations are not related to the expansions of any other fields, and so we are free to impose whatever self-consistent boundary conditions we wish on these fields. We can make an independent choice of boundary condition for each independent component of χ^{rst}, with the boundary conditions for the corresponding $\mathcal{S}^{rstI}$ and $\mathcal{P}^{rstI}$ fields then following from the $\mathcal{N} = 2$ supersymmetry. In view of the antisymmetry and duality properties of $\mathcal{S}^{rstI}$ and $\mathcal{P}^{rstI}$, we can take α_{rst} and β_{rst} to be antisymmetric, and they should therefore satisfy

$$(\alpha_{rst}\beta_{pqu} + \beta_{rst}\alpha_{pqu})\varepsilon^{rstpqu} = 0\,, \tag{B.24}$$

where there is no summation over $r, s, \cdots, u$.

In the special case where we solve (B.24) by taking

$$\begin{aligned}
\alpha_{3pq} &= \cos\omega\,, \qquad \beta_{3pq} = \sin\omega\,, \qquad 4 \le p \le q \le 8\,,\\
\alpha_{pqr} &= \cos\omega\,, \qquad \beta_{pqr} = -\sin\omega\,, \quad 4 \le p \le q \le r \le 8\,,
\end{aligned} \tag{B.25}$$

the boundary conditions become those that would follow from the $\mathcal{N} = 3$ boundary conditions we derived earlier, by decomposing the $\mathcal{N} = 3$ triplet index I into an $\mathcal{N} = 2$ doublet and a singlet.

If $\omega = 0$, i.e. for the de Wit-Nicolai theory, we find that in addition to the $\mathcal{N} = 2$ supersymmetry-preserving boundary conditions derived above, there can also exist another new set of allowed boundary conditions, in which A_i^{rs} satisfies Neumann boundary conditions. The full set of boundary conditions in this case takes the form

$$\begin{aligned}
&A^{rI}_{(0)i}\,, \quad A^{IJ}_{(0)i} = 0\,, \quad A^{rs}_{(1)i} = 0\,, \quad \chi_+^{rIJ} = 0\,, \quad \chi_-^{rsI} = 0\,, \quad \chi_+^{rst} = 0\,,\\
&\mathcal{P}^{rsIJ}_{(2)} = 0\,, \quad \mathcal{S}^{rsIJ}_{(1)} = 0\,, \quad \mathcal{P}^{rstI}_{(1)} = 0\,, \quad \mathcal{S}^{rstI}_{(2)} = 0\,.
\end{aligned} \tag{B.26}$$

B.4 $\mathcal{N} = 1$ supersymmetry

In the $\mathcal{N} = 1$ case, we set all the supersymmetry transformation parameters to zero except ϵ^1. The SO(8) index I is split as $I = (1, r)$ where $r = 2, \ldots, 8$. The supersymmetry variation of the Dirichlet boundary conditions on the gravitino implies that

$$A^{r1}_{(0)i} = 0\,, \tag{B.27}$$

which will not impose any further condition on the spin-$\frac{1}{2}$ fields because the variation of $A^{r1}_{(0)}$ automatically vanishes under $\mathcal{N} = 1$ supersymmetry. The determination of the boundary conditions for the remaining fields requires the utilization of the supersymmetry variations of the leading terms in the Fefferman-Graham expansions for the spin-0, spin-1/2 and spin-1 fields, which are given by

$$\begin{aligned}
\delta\chi^{1rs}_+ &= 0\,, \qquad \delta\chi^{1rs}_- = 0\,, \\
\delta\chi^{rst}_+ &= -\mathcal{S}^{rst1}_{(2)}\epsilon^1_+ + 2\mathrm{i}\mathcal{P}^{rst1}_{(1)}\gamma_5\epsilon^1_- - \mathrm{i}\not{D}\mathcal{P}^{rst1}_{(1)}\gamma_5\epsilon^1_+\,, \\
\delta\chi^{rst}_- &= 2\mathcal{S}^{rst1}_{(1)}\epsilon^1_- - \mathrm{i}\mathcal{P}^{rst1}_{(2)}\gamma_5\epsilon^1_+ + \not{D}\mathcal{S}^{rst1}_{(1)}\epsilon^1_+\,, \\
\delta\mathcal{S}^{1rst}_{(1)} &= \epsilon^1_+\chi^{rst}_-, \quad \delta\mathcal{P}^{1rst}_{(1)} = -\mathrm{i}\,\epsilon^1_+\gamma_5\chi^{rst}_+\,, \\
\delta\mathcal{S}^{1rst}_{(2)} &= \left(\bar{\epsilon}^1_-\chi^{rst}_+ + \bar{\epsilon}^1_+\not{D}\chi^{rst}_+\right), \\
\delta\mathcal{P}^{1rst}_{(2)} &= -\mathrm{i}\left(\bar{\epsilon}^1_-\gamma_5\chi^{rst}_- - \bar{\epsilon}^1_+\gamma_5\not{D}\chi^{rst}_-\right), \\
\delta A^{rs}_{(0)i} &= -\Big(\cos 2\omega\ \epsilon^1_+\gamma_{(0)i}\chi^{rs1}_+ + \mathrm{i}\sin 2\omega\ \epsilon^1_+\gamma_{(0)i}\gamma_5\chi^{rs1}_-\Big) \\
\delta A^{rs}_{(1)i} &= -\Big(\mathcal{S}^{rstp}_{(1)}\epsilon^1_+\gamma_{(0)i}\chi^{tp1}_+ + \mathrm{i}\mathcal{P}^{rstp}_{(1)}\epsilon^1_+\gamma_{(0)i}\gamma_5\chi^{tp1}_- \\
&\qquad -D_i(A_{(0)})(\cos 2\omega\ \epsilon^1_+\chi^{rs1}_- + \mathrm{i}\sin 2\omega\ \epsilon^1_+\gamma_5\chi^{rs1}_+)\Big)\,,
\end{aligned} \tag{B.28}$$

where we have omitted the variations of $\mathcal{S}^{rstp}$ and $\mathcal{P}^{rstp}$, since they are dual to $\mathcal{S}^{rst1}$ and $\mathcal{P}^{rst1}$ respectively, and therefore are not independent fields. One can check that there are two sets of boundary conditions preserving $\mathcal{N} = 1$ supersymmetry. The first set, for which the vector fields all obey Dirichlet boundary conditions, is given by

$$\begin{gathered}
A^{rs}_{(0)i} = 0, \quad \cos 2\omega\ \chi^{1rs}_+ + \mathrm{i}\ \sin 2\omega\ \gamma_5\chi^{1rs}_- = 0, \quad \alpha_{rst}\chi^{rst}_+ + \mathrm{i}\,\beta_{rst}\gamma_5\chi^{rst}_- = 0\,, \\
\alpha_{rst}\mathcal{S}^{rst1}_{(2)} - \beta_{rst}\mathcal{P}^{rst1}_{(2)} = 0, \quad \beta_{rst}\mathcal{S}^{rst1}_{(1)} + \alpha_{rst}\mathcal{P}^{rst1}_{(1)} = 0\,,
\end{gathered} \tag{B.29}$$

where the totally-antisymmetric coefficients α_{rst} and β_{rst} are arbitrary constants, and the indices r, s, t are not summed over. The second set, for which the subset of vectors A^{rs}_i instead satisfy Neumann boundary conditions, takes the form

$$\begin{gathered}
A^{rs}_{(1)i} = 0\,, \quad \sin 2\omega\ \chi^{1rs}_+ - \mathrm{i}\ \cos 2\omega\ \gamma_5\chi^{1rs}_- = 0, \quad \sin 2\omega\ \chi^{rst}_+ - \mathrm{i}\ \cos 2\omega\ \gamma_5\chi^{rst}_- = 0\,, \\
\sin 2\omega\ \mathcal{S}^{rst1}_{(2)} + \cos 2\omega\ \mathcal{P}^{rst1}_{(2)} = 0\,, \quad \cos 2\omega\ \mathcal{S}^{rst1}_{(1)} - \sin 2\omega\ \mathcal{P}^{rst1}_{(1)} = 0\,.
\end{gathered} \tag{B.30}$$

We now study the possibility of imposing mixed boundary conditions (B.31) on A_i^{rs}, of the form

$$\alpha A^{rs}_{(1)i} + \beta \varepsilon_{ijk} F^{rs}_{(0)jk} = 0\,. \tag{B.31}$$

Upon using (see, for example, [33])),

$$\partial_i \epsilon_+ = \frac{1}{2}\gamma_i \epsilon_-\,, \tag{B.32}$$

the supersymmetry variation of (B.31) gives rise to terms proportional to ϵ_+ and ϵ_-. The vanishing of the terms proportional to ϵ_- requires

$$\chi_-^{rs1} = \mathrm{i}\,\nu\gamma_5\chi_+^{rs1}, \qquad \nu = \frac{\alpha\sin 2\omega + \beta\cos 2\omega}{\alpha\cos 2\omega - \beta\sin 2\omega}\,. \tag{B.33}$$

The vanishing of terms proportional to ϵ_+, on the other hand, requires

$$\begin{aligned}
&\alpha(\mathcal{S}^{rstp} - \nu\mathcal{P}^{rstp})\gamma_i\chi_+^{tp1} + \mathrm{i}\,\alpha(\nu\cos 2\omega + \sin 2\omega)\gamma_5 D_i(A_0)\chi_+^{rs1} \\
&+\beta\varepsilon^{ijk}(\cos 2\omega - \nu\sin 2\omega)\gamma_k D_j(A_0)\chi_+^{rs1} = 0\,.
\end{aligned} \tag{B.34}$$

The coefficient of the first group of terms can be made to vanish. However, the vanishing of last two groups of terms independently leads to the conditions $\omega = 0$ and $\nu = 0$. From the definition of ν in (B.33), it follows that $\beta = 0$. Thus we conclude that mixed boundary conditions are not allowed by $\mathcal{N} = 1$ supersymmetry.

The terms proportional to $D_i\chi_+$ play the key role in ruling out the mixed boundary conditions. This makes it straightforward to show that mixed boundary conditions of the form (B.31) are also forbidden by $\mathcal{N} > 1$ supersymmetry. This is due to the fact that the $D_i\chi_+$ terms from $\delta A^{rs}_{(0)i}$ and $\delta A^{rs}_{(1)i}$ are universal, with the appropriate ranges of the indices (I, r) understood. Thus our previous assumptions of no mixed boundary conditions in the $\mathcal{N} \geq 2$ analyses were justified.

References

[1] B. de Wit and H. Nicolai, $N = 8$ *supergravity*, *Nucl. Phys.* **B 208** (1982) 323 [INSPIRE].

[2] B. de Wit, H. Samtleben and M. Trigiante, *The maximal* $D = 4$ *supergravities*, *JHEP* **06** (2007) 049 [arXiv:0705.2101] [INSPIRE].

[3] G. Dall'Agata, G. Inverso and M. Trigiante, *Evidence for a family of* SO(8) *gauged supergravity theories*, *Phys. Rev. Lett.* **109** (2012) 201301 [arXiv:1209.0760] [INSPIRE].

[4] B. de Wit and H. Nicolai, *Deformations of gauged* SO(8) *supergravity and supergravity in eleven dimensions*, *JHEP* **05** (2013) 077 [arXiv:1302.6219] [INSPIRE].

[5] L. Andrianopoli, R. D'Auria, S. Ferrara, P.A. Grassi and M. Trigiante, *Exceptional* $N = 6$ *and* $N = 2$ AdS_4 *supergravity and zero-center modules*, *JHEP* **04** (2009) 074 [arXiv:0810.1214] [INSPIRE].

[6] E. Witten, SL$(2, Z)$ *action on three-dimensional conformal field theories with Abelian symmetry*, hep-th/0307041 [INSPIRE].

[7] E. Cremmer and B. Julia, *The* SO(8) *supergravity*, *Nucl. Phys.* **B 159** (1979) 141 [INSPIRE].

[8] B. de Wit, *Supergravity*, hep-th/0212245 [INSPIRE].

[9] H. Lü, Y. Pang and C.N. Pope, *An ω deformation of gauged STU supergravity*, *JHEP* **04** (2014) 175 [arXiv:1402.1994] [INSPIRE].

[10] G. Dall'Agata, G. Inverso and A. Marrani, *Symplectic deformations of gauged maximal supergravity*, *JHEP* **07** (2014) 133 [arXiv:1405.2437] [INSPIRE].

[11] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Int. J. Theor. Phys.* **38** (1999) 1113 [hep-th/9711200] [INSPIRE].

[12] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from noncritical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109] [INSPIRE].

[13] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150] [INSPIRE].

[14] I.R. Klebanov and E. Witten, *AdS/CFT correspondence and symmetry breaking*, *Nucl. Phys.* **B 556** (1999) 89 [hep-th/9905104] [INSPIRE].

[15] M. Berkooz, A. Sever and A. Shomer, *'Double trace' deformations, boundary conditions and space-time singularities*, *JHEP* **05** (2002) 034 [hep-th/0112264] [INSPIRE].

[16] E. Witten, *Multitrace operators, boundary conditions and AdS/CFT correspondence*, hep-th/0112258 [INSPIRE].

[17] A. Ishibashi and R.M. Wald, *Dynamics in nonglobally hyperbolic static space-times.* 3. *Anti-de Sitter space-time*, *Class. Quant. Grav.* **21** (2004) 2981 [hep-th/0402184] [INSPIRE].

[18] D. Marolf and S.F. Ross, *Boundary conditions and new dualities: vector fields in AdS/CFT*, *JHEP* **11** (2006) 085 [hep-th/0606113] [INSPIRE].

[19] G. Compere and D. Marolf, *Setting the boundary free in AdS/CFT*, *Class. Quant. Grav.* **25** (2008) 195014 [arXiv:0805.1902] [INSPIRE].

[20] A.J. Amsel and G. Compere, *Supergravity at the boundary of AdS supergravity*, *Phys. Rev.* **D 79** (2009) 085006 [arXiv:0901.3609] [INSPIRE].

[21] S. Hollands and D. Marolf, *Asymptotic generators of fermionic charges and boundary conditions preserving supersymmetry*, *Class. Quant. Grav.* **24** (2007) 2301 [gr-qc/0611044] [INSPIRE].

[22] A.J. Amsel and D. Marolf, *Supersymmetric multi-trace boundary conditions in AdS*, *Class. Quant. Grav.* **26** (2009) 025010 [arXiv:0808.2184] [INSPIRE].

[23] E. D'Hoker, D.Z. Freedman, S.D. Mathur, A. Matusis and L. Rastelli, *Graviton and gauge boson propagators in AdS_{d+1}*, *Nucl. Phys.* **B 562** (1999) 330 [hep-th/9902042] [INSPIRE].

[24] D.Z. Freedman, S.D. Mathur, A. Matusis and L. Rastelli, *Correlation functions in the CFT_d/AdS_{d+1} correspondence*, *Nucl. Phys.* **B 546** (1999) 96 [hep-th/9804058] [INSPIRE].

[25] P. Breitenlohner and D.Z. Freedman, *Stability in gauged extended supergravity*, *Annals Phys.* **144** (1982) 249 [INSPIRE].

[26] S.W. Hawking, *The boundary conditions for gauged supergravity*, *Phys. Lett.* **B 126** (1983) 175 [INSPIRE].

JHEP02(2015)112

[27] S. Giombi, S. Prakash and X. Yin, *A note on CFT correlators in three dimensions*, *JHEP* **07** (2013) 105 [arXiv:1104.4317] [INSPIRE].

[28] O. Aharony, O. Bergman and D.L. Jafferis, *Fractional $M2$-branes*, *JHEP* **11** (2008) 043 [arXiv:0807.4924] [INSPIRE].

[29] A. Gallerati, H. Samtleben and M. Trigiante, *The $N > 2$ supersymmetric AdS vacua in maximal supergravity*, *JHEP* **12** (2014) 174 [arXiv:1410.0711] [INSPIRE].

[30] C.N. Pope, *Consistency of truncations in Kaluza-Klein*, *Conf. Proc.* **C 841031** (1984) 429 [INSPIRE].

[31] B. de Wit and H. Nicolai, *The consistency of the S^7 truncation in $D = 11$ supergravity*, *Nucl. Phys.* **B 281** (1987) 211 [INSPIRE].

[32] B.E.W. Nilsson and C.N. Pope, *Hopf fibration of eleven-dimensional supergravity*, *Class. Quant. Grav.* **1** (1984) 499 [INSPIRE].

[33] H. Lü, C.N. Pope and P.K. Townsend, *Domain walls from anti-de Sitter space-time*, *Phys. Lett.* **B 391** (1997) 39 [hep-th/9607164] [INSPIRE].